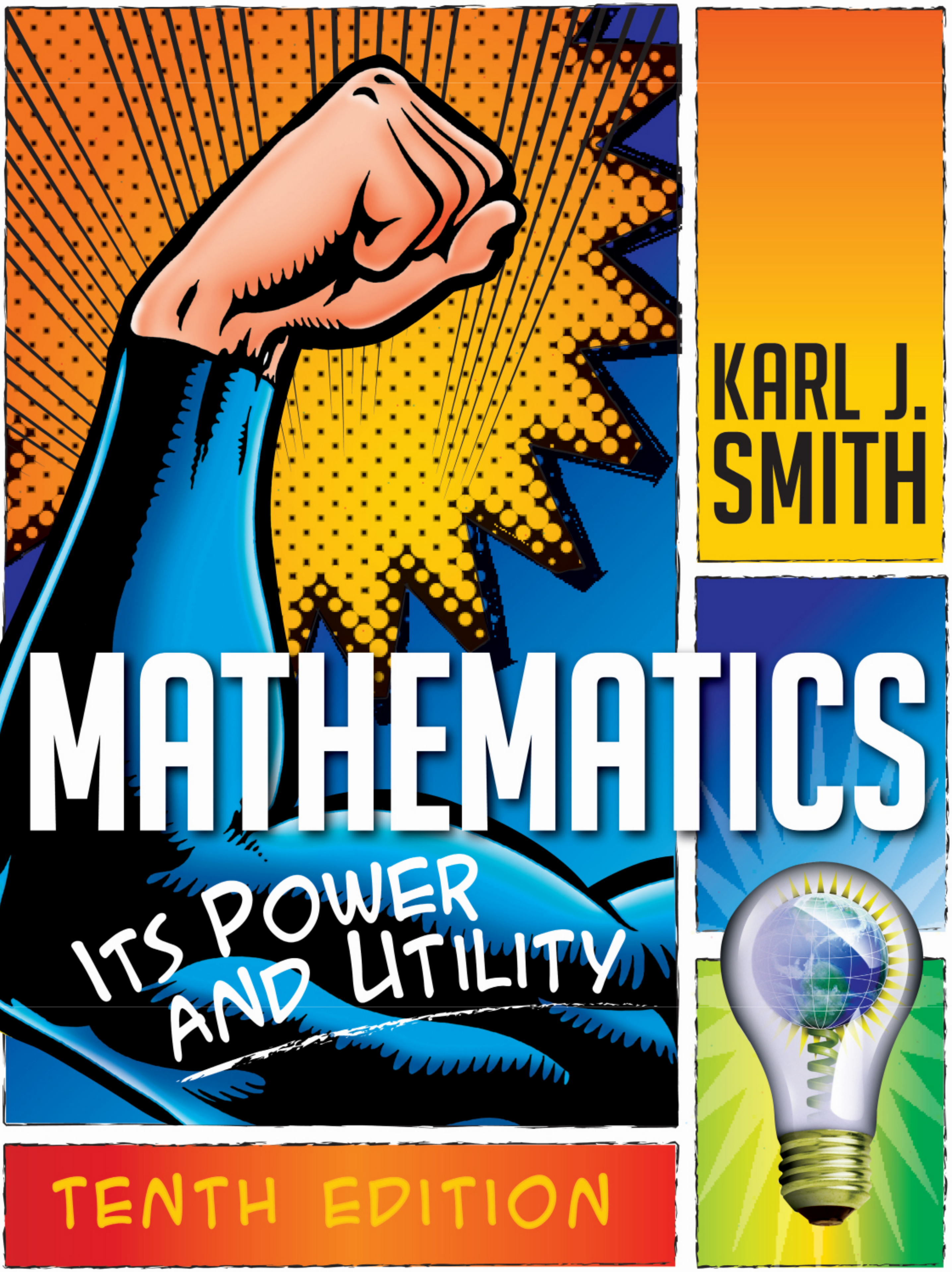
KARL J.
SMITH
MATHEMATICS
ITS POWER
AND UTILITY
TENTH EDITION

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PART 1

FOUNDATIONS

The POWER of Math

Historically, the prime value of mathematics has been that it enables us to answer basic questions about our physical world, to comprehend the complicated operations of nature, and to dissipate much of the mystery that envelops life. The simplest arithmetic, algebra, and geometry suffice to determine the circumference of the earth, the distances to the moon and the planets, the speeds of sound and light, and the reasons for eclipses of the sun and moon. But the supreme value of mathematics, insofar as understanding the world about us is concerned, is that it reveals order and law where mere observation shows chaos. . . .

Morris Kline

Mathematics: An Introduction to Its Spirit and Use, San Francisco: W. H. Freeman, 1979, p. 1.

The first part of this book, The Power of Mathematics, attempts to develop an appreciation for mathematics by displaying the intrinsic **power** of the subject. We will begin by looking at some of the causes and effects of *math anxiety*. We take natural steps—small at first and then a little larger as you gain confidence—to review and learn about calculators, fractions, percents, algebra, equations, metrics, and geometry.

Most people view mathematics as a series of techniques that are useful only to the scientist, the engineer, or the specialist. In fact, the majority of our population could be classified as math-avoiders, who consider the assertion that mathematics can be creative, beautiful, and significant to be not only an “impossible dream,” but also something they don’t even want to discuss.

At each turn of the page, I hope you will find something new and interesting. I want you to participate and become involved with the material. I want you to experience what I mean when I speak of the *beauty* of mathematics. I hope you are now ready to begin your study of a new course. I wish you success.

ANTICIPATE

- Overview each chapter before you begin.
- Look over the chapter contents.
- Important terms are listed on page 69.
- Essential ideas are listed on page 2.
- Learning outcomes are listed on page 69.
- Attitude—you may have some unpleasant memories surrounding arithmetic in elementary school. In acknowledging that this is true for many people, we begin by discussing math anxiety. What is the message? Relax . . . it will not be as bad as you might think!

Kevin Curry/Getty Images

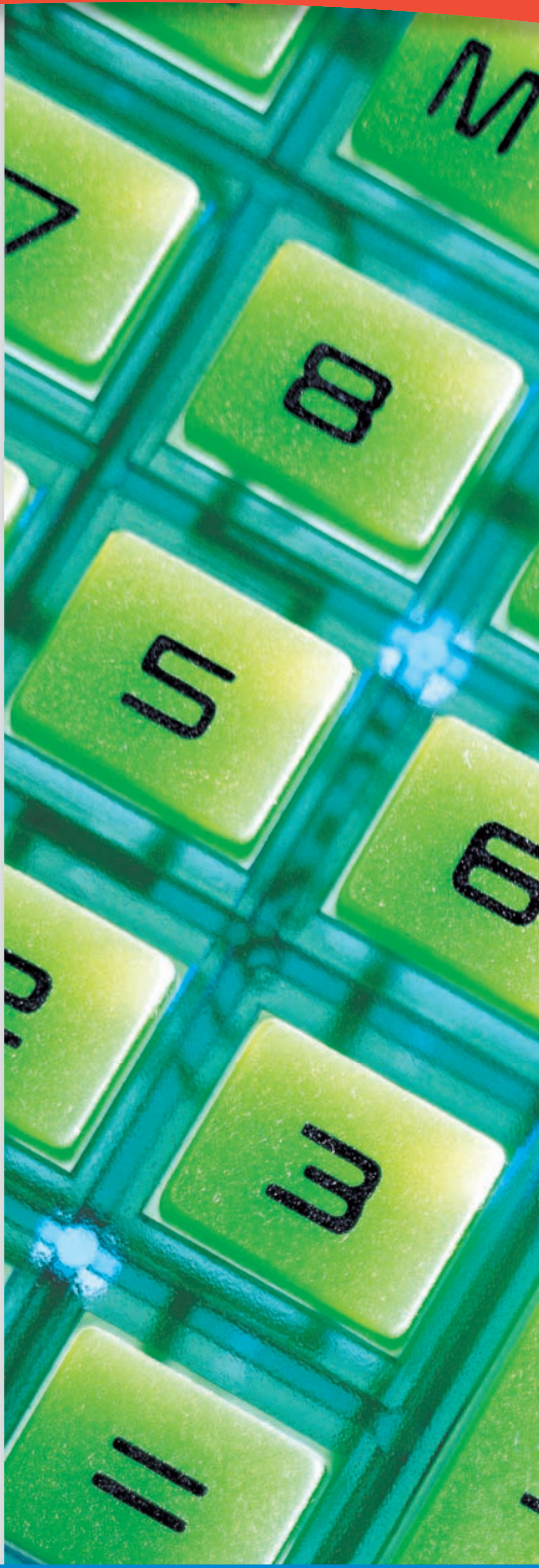
ESSENTIAL IDEAS

To help you succeed in this course, we have included these essential ideas as problems at the beginning of each section.

Section	Essential Ideas	Problems
1.1	Road sign warnings	Problems 3-6
	Behavior for success in this course	Problems 7-14
1.2	Order of operations	Problem 3
	Distributive property	Problem 4
1.3	Division by zero	Problem 1
	Meaning of fractions	Problems 2, 3, and 4
	Place value meanings	Problem 5
1.4	Rounding place digit	Problem 3
	Process of rounding	Problem 4
1.5	Definition of exponent, terminology of power notation	Problems 3, 7-10
	Scientific notation	Problem 4
	Prime factorization	Problem 5
	Distinguish exponent and EE calculator keys	Problem 6
1.6	Fundamental property of fractions	Problem 2
	Reducing fractions	Problems 3 and 4
	Multiplying and dividing fractions	Problems 5-6
	Identifying a terminating decimal	Problem 7
	Changing decimals to fractions	Problem 8
1.7	Adding and subtracting fractions	Problems 1, 2, and 6
	Extended order of operations	Problem 5
1.8	Meaning of Hindu-Arabic numerals	Problems 1, 2, 7, and 8
	Contrast number/numeral	Problem 5
	Expanded notation	Problem 6
1.9	Changing from one base to another	Problems 5-8

CHAPTER 1

Arithmetic, Calculators, and Problem Solving



The only way I can distinguish proper from improper fractions is by their actions.

OGDEN NASH

Stockbyte/Getty Images.

1.1 Math Anxiety

The POWER of Math

"Cole! What classes are you taking this semester?" asked Louise, after spotting her friend in the hallway.

"I'm taking English and history, and my advisor told me I need to take this algebra course," answered Cole, rolling his eyes. "I can't stand math. I doubt I'll make it through the first two weeks."

Louise chuckled. "Well, who's teaching your course?"

"Professor Hunter, why do you ask?" replied Cole.

"Don't worry about it, my friend. Dr. Hunter really explains the concepts well. Just don't be afraid to ask questions, and you'll be fine."

Cole smiled. "Maybe it won't be so bad, then. Sometimes, all I need is a little guidance."

There are many reasons for reading a book, but the best reason is because you want to read it. Although you are probably reading this first page because your instructor requested that you do so, it is my hope that in a short while, you will be reading this book because you want to read it.

STOP You might have skipped the box titled "The Power of Math," but these boxes are important! The mathematics you will learn in this course is not about textbook problems, homework, or even getting a good grade. The mathematics you will learn in this course is about giving **you** some necessary **life skills**. Start each section by reading the material in these boxes and asking yourself the question "What does this have to do with me?"

Peanuts © Used by permission of Universal Uclick. All rights reserved.

Do you think that you are reasonably successful in other subjects but are unable to do math? Do you make career choices based on avoidance of mathematics courses? If so, you have *math anxiety*. If you reexamine your negative feelings toward mathematics, you can overcome them. In this book, I'll constantly try to help you overcome these feelings.

This book was written for people who are math-avoiders, people who think they can't work math problems, and people who think they are never going to use math. Do you see yourself making any of the statements shown in Figure 1.1?

Figure 1.1 Math quotations—sound familiar?

Sheila Tobias, an educator, feminist, and founder of an organization called Overcoming Math Anxiety, has become one of our nation's leading spokespersons on math anxiety. She is not a mathematician; in fact, she describes herself as a math-avoider. She has written a book titled *Overcoming Math Anxiety* (New York: W. W. Norton & Company, 1978; available in paperback). I recommend this book to anyone who has ever said, "I'm no good at numbers." In this book, she describes a situation that characterizes anxiety (p. 45):

> Paranoia comes quickly on the heels of the anxiety attack. "Everyone knows," the victim believes, "that I don't understand this. The teacher knows. Friends know. I'd better not make it worse by asking questions. Then everyone will find out how dumb I really am." This paranoid reaction is particularly disabling because fear of exposure keeps us from constructive action. We feel guilty and ashamed, not only because our minds seem to have deserted us but because we believe that our failure to comprehend this one new idea is proof that we have been "faking math" for years.

The reaction described in this paragraph sets up a vicious cycle. The more we avoid math, the less able we feel; and the less able we feel, the more we avoid math. The cycle can also work in the other direction. What do you like to do? Chances are, if you like it, you do it. The more you do something, the better you become at it. In fact, you've probably thought, "I like to do it, but I don't get to do it as often as I'd like to." This is the normal attitude toward something you like to do. In this book, I attempt to break the negative cycle concerning math and replace it with a positive cycle. However, I will need your help and willingness to try.

The central theme in this book is problem solving. Through problem solving, I'll try to dispel your feelings of panic. Once you find that you are capable of doing mathematics, we'll look at some of its foundations and uses. There are no prerequisites for this book; and as we progress through the book, I'll include a review of the math you never quite learned in school—from fractions, decimals, percents, and metrics to algebra and geometry. I hope to answer the questions that perhaps you were embarrassed to ask.

At the end of each section in this book is a problem set. This first problem set is built around 12 math myths. These myths are in another book on math anxiety, *Mind Over Math*, by Stanley Kogelman and Joseph Warren (New York: Dial Press, 1978), which I highly recommend. These commonly believed myths have resulted in false impressions about how math is done, and they need to be dispelled.

Math Anxiety Bill of Rights
by Sandra L. Davis

1. I have the right to learn at my own pace and not feel put down or stupid if I'm slower than someone else.
2. I have the right to ask whatever questions I have.
3. I have the right to need extra help.
4. I have the right to ask a teacher or TA for help.
5. I have the right to say I don't understand.
6. I have the right not to understand.
7. I have the right to feel good about myself regardless of my abilities in math.
8. I have the right not to base my self-worth on my math skills.
9. I have the right to view myself as capable of learning math.
10. I have the right to evaluate my math instructors and how they teach.
11. I have the right to relax.
12. I have the right to be treated as a competent adult.
13. I have the right to dislike math.
14. I have the right to define success in my own terms.

From Tobias, Sheila *Overcoming Math Anxiety* (pp.236–237). Copyright © 1993, 1978 by Sheila Tobias. Reprinted by permission of W.W. Norton & Company.

Throughout this book, I will take off my author hat and put on my teacher hat to write you notes about the material in the book. These notes will explain steps or give you hints on what to look for as you are reading the book. These notes are printed in this font.

I have included road signs throughout this book to help you successfully get through it.

Hints for Success

Mathematics is different from other subjects. One topic builds on another, and you need to make sure that you understand *each* topic before progressing to the next one.

You must make a commitment to attend each class. Obviously, unforeseen circumstances can come up, but you must plan to attend class regularly. Pay attention to what your teacher says and does, and take notes. If you must miss class, write an outline of the text corresponding to the missed material, including working out each text example on your notebook paper.

You must make a commitment to daily work. Do not expect to save up and do your mathematics work once or twice a week. It will take a daily commitment on your part, and you will find mathematics difficult if you try to "get it done" in spurts. You could not expect to become proficient in tennis, soccer, or playing the piano by practicing once a week, and the same is true of mathematics. Try to schedule a regular time to study mathematics each day.

You must read the text carefully. Many students expect to get through a mathematics course by beginning with the homework problems, then reading some examples, and reading the text only as a desperate attempt to find an answer. This procedure is backward; do your homework only *after* reading the text.

You must ask questions. Part of learning mathematics involves frustration. Don't put off asking questions when you don't understand something or if you feel an anxiety attack coming. STOP, and put this book aside for a while. Talk to your instructor, or call me. My telephone number is

(707) 829-0606

When you see the stop sign, you should stop for a few moments and study the material next to the stop sign. It is a good idea to memorize this material.

When you see the caution sign, you should make a special note of the material next to the caution sign because it will be used throughout the rest of the book.

When you see the yield sign, it means that you need to remember only the stated result and that the derivation is optional.

When you see the bump sign, some unexpected or difficult material follows, and you will need to slow down to understand the discussion.

I care about your progress with the course, and I'd like to hear your reactions to this book. I can be reached by e-mail at

smithkjs@mathnature.com

Direct Your Focus

Read the following story. No questions are asked, but try to imagine yourself sitting in a living room with several others who share your feelings about math. Your job is to read the story and make up a problem you know how to solve from any part of the story. You should have a pencil and paper, and you can have as much time as you want. Nobody will look at what you are doing, but I want you to keep track of your feelings as your read the story and follow the directions.

Try reading this orally to your class, and then do this subsection of the text as a classroom exercise.

> On the way to the market, which is 12 miles from home, I stopped at the drugstore to pick up a get-well card. I selected a series of cards with puzzles on them. The first one said, "A bottle and a cork cost $1.10, and the bottle is a dollar more than the cork. How much is the bottle and how much is the cork?" I thought that would be a good card for Joe, so I purchased it for $1.75, along with a six-pack of cola for $2.79. The total bill was $4.81, which included 6% sales tax. My next stop was the market, which was exactly 3.4 miles from the drugstore. I bought $15.65 worth of groceries and paid with a $20 bill. I deposited the change in a charity jar on the counter and left the store. On the way home, I bought 8.5 gallons of gas for $22.10. Because I had gone 238 miles since my last fill-up, I was happy with the mileage on my new car. I returned home and made myself a ham and cheese sandwich.

Have you spent enough time on the story? Take some time to reread it (spend at least 10 minutes with this exercise). Now write down a math question, based on this story, that you could answer without difficulty. Can you summarize your feelings? If my experiences in doing this exercise with my students apply to you, I would guess that you encountered some difficulty, some discomfort, perhaps despair or anger, or even indifference. Most students tend to focus on the more difficult questions (perhaps a miles-per-gallon problem) instead of following the directions to formulate a problem that will give you no difficulty.

How about the question "What is the round-trip distance from home to market and back?" *Answer:*

$$2 \times 12 \text{ miles} = 24 \text{ miles}$$

You say, "What does this have do with mathematics?" The point is that you need to learn to *focus on what you know* rather than on what you do not know. You may surprise yourself with the amount that you do know and what *you* can bring to the problem-solving process.

Math anxiety builds on focusing on what you can't do rather than on what you can do. This leads to anxiety and frustration. Do you know what is the most feared thing in our society? It is the fear of speaking in public. And the fear of letting others know you are having trouble with this problem is related to that fear of speaking in public.

If you focus on a problem that is too difficult, you will be facing a blank wall. This applies to all hobbies or subjects. If you play tennis or golf, has your game improved since you started? If you don't play these games, how do you think you would feel trying to learn in front of all your friends? Do you think you would feel foolish?

Mathematicians don't start with complicated problems. If a mathematician runs into a problem that she can't solve, she will probably rephrase the problem as a simpler, related problem that she can't solve. This problem is, in turn, rephrased as yet a simpler problem, and the process continues until the problem is manageable, and she has a problem that she *can* solve.

Writing Mathematics

The fundamental objective of education always has been to prepare students for life. A measure of your success with this book is a measure of its usefulness to you in your life. What are the basics for your knowledge "in life"? In this information age with access to a world of knowledge on the Internet, we still would respond by saying that the basics remain "reading, 'riting, and 'rithmetic." As you progress through the material in this book, we will give you opportunities to read mathematics and to consider some of the great ideas in the history of civilization, to develop your problem-solving skills ('rithmetic), and to communicate mathematical ideas to others ('riting). Perhaps you think of mathematics as "working problems" and "getting answers," but it is so much more. Mathematics is a way of thought that includes all three Rs; and to strengthen your skills you will be asked to communicate your knowledge in written form.

Journals

Darren Baker/Shutterstock.com

To begin building your skills in writing mathematics, you might keep a journal summarizing each day's work. Keep a record of your feelings and perceptions about what happened in class. How long did the homework take? What time of the day or night did you spend working and studying mathematics? What is the most important idea from the day's lesson? To help you with your journals, you will find problems in this text designated **IN YOUR OWN WORDS**. (For example, look at Problems 15-26 in Problem Set 1.1.) There are no right answers or wrong answers to this type of problem, but you are encouraged to look at these for ideas of what you might write in your journal.

Journal ideas

Write in your journal every day.

Include important ideas.

Include new words, ideas, formulas, or concepts.

Include questions that you want to ask later.

If possible, carry your journal with you so you can write in it anytime you get an idea.

Reasons for keeping a journal

It will record ideas you might otherwise forget.

It will keep a record of your progress.

If you have trouble later, it may help you diagnose areas for change or improvement.

It will build your writing skills.

Problem Set 1.1

The POWER of Math

1. **IN YOUR OWN WORDS*** What is your math history?Briefly describe your chronological history in terms of the negative and positive experiences you've had with math. Include your earliest memories as well as memories of how your teachers and your family influenced you in math. Describe how your family members approached math, and describe their attitude toward your math ability. Include a description of how you've dealt with recent situations involving math in other classes, on the job, or in daily life situations. End with a discussion of how math could help you in accomplishing your educational objectives, in earning more money, in choosing a career, or in any other aspect of your life.
2. **IN YOUR OWN WORDS** We printed a "Math Anxiety Bill of Rights," by Sandra L. Davis, in this section. Turn in your own list of "Math Rights."

LEVEL 1 Essential Ideas

In Problems 3-6*, describe the meaning of each of the symbols, which you will find throughout the book.*

3. 

4. 

5. 

6. 

IN YOUR OWN WORDS *In this section, there are some hints for success. Discuss the statements in Problems* 7-12*, which refer to these hints.*

7. You must make a commitment to attend each class. Do you think this means each and every class?
8. If it is impossible to attend class (because of a doctor visit, for example), it is acceptable to skip over the day's activities and pick up with the material when you return.
9. If it is impossible to turn in one or two assignments (because of illness, for example), it is acceptable to skip those assignments and pick up with the material when you return.
10. You must make a commitment to do daily work. Do you think this means turning in each and every assignment?
11. You must read the text carefully. Do you think this means reading every word?
12. You must ask questions.
13. **IN YOUR OWN WORDS** Some hints for success are listed on page 6. Are there any other strategies for success that you have used that work for you?
14. **IN YOUR OWN WORDS** Consider the hints for success on page 6. Is there any reason why you don't think one of these strategies for success will work for you?

LEVEL 2 Drill and Practice

IN YOUR OWN WORDS *In Problems* 15-26*, comment on each math myth.*

15. Myth 1: Men are better than women in math.
16. Myth 2: Math requires logic, not intuition.
17. Myth 3: You must always know how you got the answer.
18. Myth 4: Math is not creative.
19. Myth 5: There is a best way to do a math problem.
20. Myth 6: It's always important to get the answer exactly right.
21. Myth 7: It's bad to count on your fingers.
22. Myth 8: Mathematicians do problems quickly, in their heads.
23. Myth 9: Math requires a good memory.
24. Myth 10: Math is done by working intensely until the problem is solved.
25. Myth 11: Some people have a "math mind" and some don't.
26. Myth 12: There is a magic key to doing math.
27. **IN YOUR OWN WORDS** Summarize your math experiences in elementary school.
28. **IN YOUR OWN WORDS** Summarize your math experiences in high school.
29. **IN YOUR OWN WORDS** Summarize your feelings today about this course.
30. **IN YOUR OWN WORDS** Describe a good experience you have had concerning mathematics.
31. **IN YOUR OWN WORDS** Describe an unpleasant experience you have had concerning mathematics.

*From AREM, *Conquering Math Anxiety: A Self-Help Workbook, 2/e*. Copyright © 2003 Brooks/Cole. Reproduced by permission Cengage Learning, Inc. www.cengage.com/permissions

32. **IN YOUR OWN WORDS** There are eight math quotations in Figure 1.1. Which one best describes your feelings about math today? Give reasons.

33. **IN YOUR OWN WORDS** There are eight math quotations in Figure 1.1. Which one is least like your feelings about math today? Give reasons.

34. **IN YOUR OWN WORDS** Do see any possible barriers that will inhibit your success in this course. List them for this problem, and then discuss them with at least one individual.

35. **IN YOUR OWN WORDS** Make a list of positive forces that may help you with your success in this course.

36. **IN YOUR OWN WORDS** Provide a list of people on whom you might call to help you with this course.

LEVEL 2 Applications

Symbols similar to the ones used in this book are common on the highways, and now we are seeing "universal" nonverbal symbols for other things. Problems 37-42 *show universal traffic signs. See whether you can guess what each symbol means.*

37.

38.

39.

40.

41.

42.

In Problems 43-48*, symbols that you might see in an airport are shown. See whether you can guess what each symbol means.*

43.

44.

45.

46.

47.

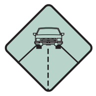
48.

In Problems 49-54*, symbols used in the classified section of a newspaper are shown. Match each symbol to one of the following classifications:*

A. Announcements
B. Employment
C. Rentals
D. Merchandise
E. Recreation
F. Transportation medium

49.

50.



51.

52.

53.

54.

IN YOUR OWN WORDS *In Problems* 55-56*, read the story, and make up a problem you can solve from some part of the story.*

55. *Yesterday, I purchased five calves at the auction for $95 each, so my herd now consists of nineteen cows, one bull, twenty-six steers, and thirteen calves. The auction yard charged me $35 to deliver the calves to my ranch, but I figured it was a pretty good deal since I live 42 miles from the auction yard. Today, twelve tons of hay were delivered, and I paid $780 for it plus $10 a ton delivery charge. Yes, sir, if my crops do well, this will be a very good year.*

56. *Tickets for the concert go on sale for $65 each tomorrow; each person is allowed to purchase no more than four tickets. I really don't mind paying for the tickets, but why do they need to add a $3-per-ticket service charge? I am going to pick Jane up at 5:00 A.M. so we can get in line early; the only problem is that Jane lives 23 miles from me and the ticket office is only 4 miles from my house. That means that we will not be able to line up before 6:00 A.M. Do you think that is early enough?*

57. **IN YOUR OWN WORDS** Use Problem 55 to make up a second problem.

58. **IN YOUR OWN WORDS** Use Problem 56 to make up a second problem.

59. **IN YOUR OWN WORDS** Describe some of your feelings as you worked Problem 55.

60. **IN YOUR OWN WORDS** Describe some of your feelings as you worked Problem 56.

1.2 Formulating the Problem

The POWER of Math

"Check it out," explains Alfonso. "If you divide these two numbers, you can finish the problem."

"Sweet—it's all starting to make sense!" Jerry proclaims with a sheepish grin. "Is it cool if I borrow your calculator, Teresa?"

"We need a calculator for this class?" Teresa asks.

Alfonso shakes his head. "Get with the program, Teresa. You should always keep one handy."

In this section, you will learn about formulating the problem as well as about some of what we call *elementary operations*. You will also be encouraged to use a calculator for working problems in this book.

In mathematics, we generally focus our attention on some particular sets of numbers. The simplest of these sets is the set we use to count objects, and it is the first set that a child learns. It is called the set of *counting numbers* or *natural numbers*.*

Natural Number and Whole Numbers

The set of numbers $\{1, 2, 3, 4, \ldots\}$ is called the set of **counting numbers** or the set of **natural numbers**. If the number zero is included, then the set $\{0, 1, 2, 3, 4, \ldots\}$ is called the set of **whole numbers**.

Order of Operations

Addition, subtraction, multiplication, and division are called the **elementary operations** for the whole numbers, and it is assumed that you understand these operations. However, certain agreements in dealing with these operations are necessary. Consider this arithmetic example:

$$\textit{Find: } 2 + 3 \times 4$$

There are two possible approaches to solve this problem:

$$\textit{Left to right: } 2 + 3 \times 4 = 5 \times 4 = 20$$

$$\textit{Multiplication first: } 2 + 3 \times 4 = 2 + 12 = 14$$

Because there are different results to this arithmetic example, it is necessary for us all to agree on one method or the other. At this point, you might be thinking, "Why on earth would I start at the right and do the multiplication first?" Consider the following example.

*In mathematics, we use the word *set* as an undefined term. Although sets are discussed in Chapter 8, we assume that you have an intuitive idea of the word *set*. It is used to mean a collection of objects or numbers. Braces are used to enclose the elements of a set, and three dots (called ellipses) are used to indicate that some elements are not listed. We use three dots only if the elements not listed are clear from the given numbers.

EXAMPLE 1 Order of operations

Suppose that you sold a $2 benefit ticket on Monday and three $4 tickets on Tuesday. What is the total amount you collected?

Solution (sales on monday) + (sales on tuesday) = (total sales)

$$\$2 + 3 \times \$4 = \text{(total sales)}$$

Correct	*Not Correct*
Multiplication first:	Left to right:
$2 + 3 \times 4 = 2 + 12$	$2 + 3 \times 4 = 5 \times 4$
$= 14$	$= 20$

Do you see why the correct result to Example 1 requires multiplication before addition?

Many of you may use a calculator to help you work problems in this book. If you do use a calculator, it is important that you use a calculator that carries out the correct order of operations, as we illustrate with the next example.

EXAMPLE 2 Order of operations using a calculator

In Example 1, we agreed that the correct simplification of the arithmetic problem

$$2 + 3 \times 4$$

is 14. Illustrate the correct buttons to press on a calculator, and check with the calculator you will use in this class to make sure you obtain the correct result.

Solution You want the calculator you use in this class to accept the input of arithmetic problems in the same fashion as you would write it on your paper. Thus, the correct sequence of keys to press is

$$\boxed{2}\ \boxed{+}\ \boxed{3}\ \boxed{\times}\ \boxed{4}\ \boxed{=}$$

CAUTION

If you are using a calculator with this book, be sure you actually check out this answer.

Some calculators use a key labeled $\boxed{\text{ENTER}}$ instead of the equal sign. The correct answer is 14, but some calculators will display the incorrect answer of 20. What is going on with a calculator that gives the answer 20? Such a calculator works from left to right without regard to the order of operations. For this book, you want to have a calculator that has been programmed to use the order of operations correctly.

If the operations are mixed, we agree to do multiplication and division *first* (from left to right) and *then* addition and subtraction from left to right.

EXAMPLE 3 Order of operations

Perform the indicated operations.

a. $7 + 2 \times 6$ **b.** $10 - 6 \div 2$ **c.** $6 \div 2 \times 3 + 4$
d. $3 \times 5 + 2 \times 5$ **e.** $1 + 3 \times 2 + 4 - 3 + 6 \times 3$

Solution

a. $7 + 2 \times 6 = 7 + 12$ Multiplication first
$= 19$ Addition next

b. $10 - 6 \div 2 = 10 - 3$ Division first
$= 7$ Addition next

c. $6 \div 2 \times 3 + 4 = 3 \times 3 + 4$
$= 9 + 4$ Multiplication/division first (left to right)
$= 13$ Addition/subtraction (left to right)

d. $3 \times 5 + 2 \times 5 = 15 + 10$ Multiplication first
$= 25$ Addition next

e. $1 + 3 \times 2 + 4 - 3 + 6 \times 3 = 1 + 6 + 4 - 3 + 18$
$= 7 + 4 - 3 + 18$
$= 11 - 3 + 18$
$= 8 + 18$
$= 26$

If the order of operations is to be changed from this agreement, then parentheses are used to indicate this change.

EXAMPLE 4 Order of operations with parentheses

Perform the indicated operations.

a. $10 + 6 + 2$ **b.** $10 + (6 + 2)$ **c.** $10 - 6 - 2$ **d.** $10 - (6 - 2)$

e. Show the buttons you would press to work part **d** with a calculator.

Solution

a. $10 + 6 + 2 = (10 + 6) + 2$ Work from left to right; parentheses are understood.
$= 16 + 2$
$= 18$

b. $10 + (6 + 2) = 10 + 8$ Parentheses change the order of operations.
$= 18$

Notice that parts **a** and **b** illustrate different problems that have the same answer. This is not the case with parts **c** and **d**.

c. $10 - 6 - 2 = (10 - 6) - 2$ Parentheses understood.
$= 4 - 2$
$= 2$

d. $10 - (6 - 2) = 10 - 4$ Parentheses given.
$= 6$

e. Most calculators have parentheses keys. The one labeled $\boxed{(}$ is the open parenthesis (this one comes first), and the one labeled $\boxed{)}$ is the close parenthesis (this one comes last). Locate these keys on your calculator. Also note that the operations keys are usually found at the right side. These are labeled $\boxed{\div}$ $\boxed{\times}$ $\boxed{-}$ $\boxed{+}$. Do not confuse subtraction $\boxed{-}$ with other keys labeled $\boxed{(-)}$ or $\boxed{+/-}$. The correct sequence of buttons to press for this example is

$$\boxed{10}\ \boxed{-}\ \boxed{(}\ \boxed{6}\ \boxed{-}\ \boxed{2}\ \boxed{)}\ \boxed{=}$$

Check the output of your calculator to make sure it gives the result shown in part **d**, namely, 6.

We can now summarize the correct **order of operations**, including the use of parentheses.

Order of Operations

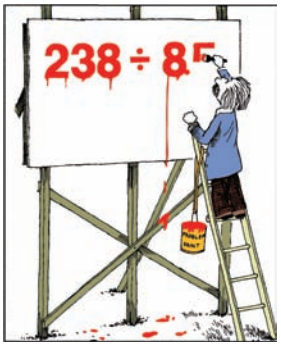
In doing arithmetic with mixed operations, we agree to proceed by following these steps:

Step 1 Parentheses first
Step 2 Multiplications and divisions, reading from left to right
Step 3 Additions and subtractions, reading from left to right

STOP Take a few minutes to memorize this order of operations agreement.

You can remember this by using the mnemonic "**P**lease **M**ind **D**ear **A**unt **S**ally." The first letters will remind you of "**P**arentheses, **M**ultiplication and **D**ivision, **A**ddition and **S**ubtraction." However, if you use this mnemonic, remember that the order is *left to right* for pairs of operations: multiplication and division (as they occur) and *then* addition and subtraction (as they occur).

EXAMPLE 5 Mixed operations



Perform the indicated operations.

a. $12 + 9 \div 3$ **b.** $(12 + 9) \div 3$

c. $2 \times 15 + 9 \div 3 - 7 \times 2$ **d.** $2 \times (15 + 9) \div 3 - 7 \times 2$

Solution

a. $12 + 9 \div 3 = 12 + 3$ Multiplications/divisions before additions/subtractions

$= 15$ Additions/subtractions as they occur from left to right

b. $(12 + 9) \div 3 = 21 \div 3$ Parentheses first

$= 7$ Division next

c. $2 \times 15 + 9 \div 3 - 7 \times 2 = 30 + 3 - 14$ Multiplications/divisions

$= 33 - 14$ Additions/subtractions

$= 19$

d. $2 \times (15 + 9) \div 3 - 7 \times 2 = 2 \times 24 \div 3 - 7 \times 2$ Parentheses first

$= 48 \div 3 - 7 \times 2$ Multiplication

$= 16 - 7 \times 2$ Division left to right

$= 16 - 14$ Multiplication

$= 2$ Subtraction

Simplify a Numerical Expression

The expressions in Example 5 are called *numerical expressions*. A **numerical expression** is one or more numbers connected by valid mathematical operations (such as *addition, subtraction, multiplication*, or *division*). To **simplify a numerical expression** means to carry out all the operations, according to the order of operations, and to write the answer as a single number.

EXAMPLE 6 Simplifying a numerical expression

Simplify $2[56 - 4(2 + 8)] + 5 - 4(2 + 1)$.

Solution If there are parentheses inside parentheses (or brackets), the order of operations directs us to begin with the interior set of parentheses (shown in color):

$2[56 - 4(2 + 8)] + 5 - 4(2 + 1) = 2[56 - 4(10)] + 5 - 4(2 + 1)$ Interior parentheses first

$= 2[56 - 40] + 5 - 4(3)$ Parentheses next

$= 2[16] + 5 - 12$ All parentheses should be completed before continuing; note that the brackets are used as parentheses.

$= 32 + 5 - 12$ Multiplications/divisions next

$= 25$ Additions/subtractions

Pólya's Problem-Solving Method

The model for problem solving that we will use was first published in 1945 by the great, charismatic mathematician George PÓlya. His book *How to Solve It* (Princeton University Press, 1973) has become a classic. In PÓlya's book, you will find this problem-solving model as well as a treasure trove of strategy, know-how, rules of thumb, good advice, anecdotes, history, and problems at all levels of mathematics. His problem-solving model is as follows.

Problem Solving

The problem-solving guidelines known as **Pólya's method** sets forth the following steps:

Step 1 *Understand the problem*. Ask questions, experiment, or otherwise rephrase the question in your own words.

Step 2 *Devise a plan*. Find the connection between the data and the unknown. Look for patterns, relate to a previously solved problem or a known formula, or simplify the given information to give you an easier problem.

Step 3 *Carry out the plan*. Check the steps as you go.

Step 4 *Look back*. Examine the solution obtained. In other words, check your answer.

We will use this procedure as we develop the problem-solving idea throughout the book. However, we are not ready to tackle real problem solving; we need to start at the beginning with small steps. One of these small steps is developing skill in **translating** from English into math symbolism. The term **sum** is used to indicate the result obtained from addition, **difference** for the result from subtraction, **product** for the result of a multiplication, and **quotient** for the result of a division. When a problem involves mixed operations, it is classified as a sum, difference, product, or quotient according to the *last* operation performed, when using the order-of-operations agreement.

Historical Note

Karl Smith Library

George Pólya

(1887–1985)

George Pólya was one of the most beloved mathematicians of the 20th century. His research and winning personality earned him a place of honor not only among mathematicians, but among students and teachers as well. His discoveries spanned an impressive range of mathematics, real and complex analysis, probability, combinatorics, number theory, and geometry. Pólya's How to Solve It has been translated into 15 languages. His books have a clarity and elegance seldom seen in mathematics, making them a joy to read. For example, here is his explanation of why he was a mathematician: "It is a little shortened but not quite wrong to say: I thought I am not good enough for physics and I am too good for philosophy. Mathematics is in between."

EXAMPLE 7 Translating and classifying arithmetic operations

Write each verbal description in math symbols, and then use your calculator to simplify

a. The sum of the first five natural numbers
b. The product of the first five natural numbers
c. The sum of 5 and twice the number 53
d. The product of 5 and twice the number 4
e. The product of 5 and the sum of 2 and 4
f. The sum of 5 and the product of 2 and 4
g. The difference of 7 and 2 (or, equivalently, the difference of 2 from 7)

Solution

a. $1 + 2 + 3 + 4 + 5 = 15$
b. $1 \times 2 \times 3 \times 4 \times 5 = 120$
c. $5 + (2 \times 53) = 111$
d. $5 \times (2 \times 4)$ or $5 \times 2 \times 4 = 40$
e. $5 \times (2 + 4) = 5 \times 6$
$= 30$

In this book, we will generally write such a calculation as $5(2 + 4)$. The multiplication written this way is called **juxtaposition**, with the multiplication between the 5 and the parentheses understood. If you are using a calculator, check to see whether your calculator recognizes juxtaposition. Try:

No times sign required: $\boxed{5}\ \boxed{(}\ \boxed{2}\ \boxed{+}\ \boxed{4}\ \boxed{)}\ \boxed{=}$

The correct answer is 30. If your calculator does not display this result, then you will need to remember to input a times sign.

Times sign required: $\boxed{5}\ \boxed{\times}\ \boxed{(}\ \boxed{2}\ \boxed{+}\ \boxed{4}\ \boxed{)}\ \boxed{=}$

f. $5 + (2 \times 4) = 13$

The times sign *is* required for this problem, because otherwise, it would be $5 + 24 = 29$.

g. $7 - 2 = 5$

CAUTION: Pay attention to the wording on subtraction problems. Part g is not the same as $2 - 7$.

EXAMPLE 8 Comparing order of operations

Simplify, classify as a sum or a product, and then compare answers.

a. $4 \times (3 + 2)$
b. $4 \times 3 + 4 \times 2$

Solution

a. $4 \times (3 + 2) = 4 \times 5$ Parentheses
$= 20$ Multiplication

This is a product, since the last operation is multiplication.

b. $4 \times 3 + 4 \times 2 = 12 + 8$ Multiplication before addition
$= 20$

This is a sum, since the last operation is addition. Note that the answers to parts **a** and **b** are the same.

Example 8 illustrates a combined property of addition and multiplication called the **distributive property for multiplication over addition**:

$$\underset{\substack{\uparrow\\ \text{Number outside parentheses}}}{4} \times (3 + 2) = \underset{\uparrow}{4} \times 3 + \underset{\uparrow}{4} \times 2$$

Number outside parentheses is **distributed** to **each** number inside parentheses.

This property holds for all whole numbers.

EXAMPLE 9 Distributive property

Write each expression without parentheses by using the distributive property.

a. $8 \times (7 + 10)$ **b.** $3 \times (400 + 20 + 5)$

Solution

a. $8 \times (7 + 10) = 8 \times 7 + 8 \times 10$

b. $3 \times (400 + 20 + 5) = 3 \times 400 + 3 \times 20 + 3 \times 5$

The ability to recognize the difference between reasonable answers and unreasonable ones is important not just in mathematics, but whenever you are doing problem solving. This ability is even more important when you use a calculator, because pressing the incorrect key can often cause very unreasonable answers. Whenever you find an answer, you should ask yourself whether it is reasonable. How do you decide whether an answer is reasonable? One way is to **estimate** an answer. Webster's *New World Dictionary* tells us that as a verb, to *estimate* means "to form an opinion or a judgment about" or to calculate "approximately." In the *1986 Yearbook* of the National Council of Teachers of Mathematics, we find:

The broad *mathematical context* for an estimate is usually one of the following types:

A. An exact value is known, but for some reason an estimate is used.
B. An exact value is possible but is not known, and an estimate is used.
C. An exact value is impossible.

We will work on building your estimation skills throughout this book.

EXAMPLE 10 Estimation

If your salary is \$9.75 per hour, your annual salary is approximately

A. \$5,000 B. \$10,000 C. \$15,000 D. \$20,000 E. \$25,000

Solution Problem solving often requires some assumptions about the problem. For this problem, we are not told how many hours per week you work or how many weeks per year you are paid. We assume a 40-hour week, and we also assume that you are paid for 52 weeks per year.

Estimate: Your hourly salary is about \$10 per hour. A 40-hour week gives us $40 \times \$10 = \$400$ per week. For the estimate, we calculate the wages for 50 weeks instead of 52: 50 weeks yields $50 \times \$400 = \$20,000$. The answer is D.

There are two important reasons for estimation: (1) to form a reasonable opinion (as in Example 10) or (2) to check the reasonableness of an answer. If reason (1) is our motive, we should not think it necessary to follow an estimation like the one in Example 10 by direct calculation. To do so would defeat the purpose of the estimation. On the other hand, if we are using the estimate for reason (2)—to see whether an answer is reasonable—we might perform the estimate as a check on the answer for Example 10 by calculator: *Display:* 20280. The actual annual salary is \$20,280. In this case, our estimate confirms that our precise calculation does not contain a widely erroneous keying mistake.

Problem Set 1.2

The POWER of Math

1. IN YOUR OWN WORDS Are you planning on using a calculator in this class?

   IF YOUR ANSWER IS YES, answer this question:

   $$10 - 2 \times 3$$

   a. mentally **b.** by calculator

   IF YOUR ANSWER IS NO, answer this question: Do you think not having a calculator will be a hindrance to you?

2. In 2010, the Internal Revenue Service allowed a \$5,150 deduction for each dependent. In 2005, it was \$3,200. If a family has four dependents, what is the allowed deduction in 2010?

LEVEL 1 Essential Ideas

3. State the order of operations.
4. IN YOUR OWN WORDS Explain the distributive property.

LEVEL 1 Right or Wrong?

Explain what is wrong, if anything, with the statements in Problems 5-10. Explain your reasoning.

5. $4 + 6 \times 8 = 80$
6. $2 + 8 \times 10 = 82$
7. $2 \times (3 + 4) = 14$ is an example of the distributive property.
8. The correct order of operations (for an expression with no parentheses) when simplifying is first multiply, then divide, then add, then subtract.
9. The number 0 is a natural number.
10. All natural numbers are also whole numbers.

LEVEL 2 Drill and Practice

Simplify the numerical expressions given in Problems 11-24, and classify each as a sum, difference, product, or quotient.

11. **a.** $5 + 6 \times 2$
    b. $8 + 2 \times 3$
12. **a.** $20 - 4 \times 2$
    b. $10 - 5 \times 2$
13. **a.** $12 \div 6 + 3$
    b. $100 \div 10 \div 2$
14. **a.** $12 + 6 \div 3$
    b. $100 \div (10 \times 2)$
15. **a.** $15 + 6 \div 3$
    b. $16 - 6 \div 3$
16. **a.** $(15 + 6) \div 3$
    b. $(15 - 6) \div 3$
17. **a.** $4 \times 3 + 4 \times 5$
    b. $8 \times 2 + 8 \times 5$
18. **a.** $4 \times (3 + 5)$
    b. $8 \times (2 + 5)$
19. **a.** $2 + 15 \div 3 \times 5$
    b. $5 + 12 \div 3 \times 2$
20. **a.** $2 + 15 \times 3 \div 5$
    b. $5 + 12 \times 3 \div 2$
21. **a.** $(20 - 8) \div 4 \times 2 + 3$
    b. $20 - 8 \div 4 \times 2 + 3$
22. **a.** $2 + 3 \times 4 - 12 \div 2$
    b. $15 \div 5 \times 2 + 6 \div 3$
23. **a.** $2 \times 18 + 9 \div 3 - 5 \times 2$
    b. $2 \times (18 + 9) \div 3 - 5 \times 2$
24. **a.** $4 \times (12 - 8) \div 2$
    b. $4 \times 12 - 8 \div 2$

Write out the expressions in Problems 25-28, without parentheses, by using the distributive property.

25. **a.** $3 \times (4 + 8)$
    b. $7 \times (9 + 4)$
26. **a.** $8 \times (50 + 5)$
    b. $6 \times (90 + 7)$
27. **a.** $4 \times (300 + 20 + 7)$
    b. $6 \times (500 + 30 + 3)$
28. **a.** $5 \times (800 + 60 + 4)$
    b. $4 \times (700 + 10 + 5)$

Translate each of the word statements in Problems 29-36 to numerical statements.

29. The sum of three and the product of two and four
30. The product of three and the sum of two and four
31. Ten times the sum of five and six
32. Ten times the product of five and six

33. Eight times five plus ten

34. Eight times the difference of seven and five

35. Eight times the difference of nine from eleven

36. The product of the sum of three and four with the sum of five and six

Perform the indicated operations in Problems 37-52, *on your calculator, and classify each as a sum, difference, product, or quotient.*

37. $716 - 5 \times 91$

38. $143 + 12 \times 14$

39. $8 \times 14 + 8 \times 86$

40. $15 \times 27 + 15 \times 73$

41. $12 \times 63 + 12 \times 27$

42. $19 \times 250 + 19 \times 750$

43. $(18 + 2)(82 - 2)$

44. $(34 - 4)(16 + 4)$

45. $5 + 3 \times 7 + 65 - 8 \times 4$

46. $12 + 6 \times 9 - 5 \times 2 + 5 \times 14$

47. $27 \times 550 - 27 \times 450$

48. $23 \times 237 + 23 \times 763$

49. $1{,}214 - 18 \times 14 + 35 \times 8{,}121$

50. $862 + 328 \times 142 - 168$

51. $62 \times (48 - 12) + 13 \times (12 - 5)$

52. $12 \times (125 - 72) - 3 \times (18 - 3 \times 5)$

LEVEL 2 Applications

First estimate your answer in Problems 53-60, *and then calculate the exact answer.*

53. How many hours are there in 360 days?

54. How many pages are necessary to make 1,850 copies of a manuscript that is 487 pages long? (Print on one side only.)

55. If your payroll deductions are $255.83 per week and your weekly gross wages are $1,025.66, what is your net pay?

56. If your monthly salary is $1,543, what is your annual salary?

57. If you are paid $18.00 per hour, what is your annual salary?

58. If you are paid $31,200 per year, what is your hourly salary?

59. If your car gets 23 miles per gallon, how far can you go on 15 gallons of gas?

60. If your car travels 492 miles on 12 gallons of gas, what is the number of miles per gallon?

1.3 Fractions and Decimals

The POWER of Math

"Today, my professor told me that it's impossible to divide by zero," remarked Carol. "I have no idea what she means; if I can divide zero into a number and come out with nothing, then I should be able to divide by zero and get nothing too!"

"Look at it this way," chimed Tom, "if someone has six cookies and wants to divide them between you and me, we each get three. But if that person has zero people to give them to, he has no set method of dividing the cookies, so it is impossible to say what the outcome will be."

Carol nodded. "But if I was him, I'd eat all the cookies myself!"

In this section, we will not only learn about fractions and decimals, we will also see why we can't divide by zero.

In the previous section, the symbol $\div$ was used for division (for example, $10 \div 5$). However, more often a division bar is used, as in $\frac{10}{5}$.

Division By Zero—No, No, No!

EXAMPLE 1 Division notation and division by 0

Simplify (that is, perform the indicated operations).

a. $\frac{30}{6}$ **b.** $\frac{3,965}{305}$ **c.** $\frac{0}{5}$ **d.** $\frac{5}{0}$ **e.** $\frac{10+8}{7+2}$ **f.** $\frac{10}{3}$

Solution

a. $\frac{30}{6} = 5$ *Check by multiplication:* $6 \times 5 = 30$

b. If the division is lengthy, as in the case of $\frac{3,965}{305}$, you may need to do long division or use a calculator:

$$\begin{array}{r} 13 \\ 305\overline{)3965} \\ \underline{305} \\ 915 \\ \underline{915} \\ 0 \end{array}$$

Check: $305 \times 13 = 3,965$

Thus, $\frac{3,965}{305} = 13$.

c. $\frac{0}{5} = 0$ *Check by multiplication:* $5 \times 0 = 0$

STOP Remember, division by 0 is impossible.

d. $\frac{5}{0}$ To do this division, you would need to find a number such that, when it is multiplied by zero, the result is 5. There is no such number. For this reason, we say **division by zero is impossible**.

e. **The division bar also acts like a grouping symbol**. This division problem implies that we are to find the sum of 10 and 8 and then divide that answer by the sum of 7 and 2:

CAUTION Don't forget that the division bar in a fraction is used as a grouping symbol.

$$\frac{10+8}{7+2} = \frac{18}{9} = 2$$

You might wish to simplify this expression by using a calculator. When entering expressions such as this, if the one on top (or the one on the bottom) is more than a single number, then you must insert parentheses:

CAUTION

By calculator, think of this as $\frac{10+8}{7+2}$:

(10 + 8) ÷ (7 + 2) =

f. $\frac{10}{3}$ There is no answer to this division in the set of whole numbers.

Definition of Fraction

The reason there is no answer to $\frac{10}{3}$ in the set of whole numbers in part **f** of Example 1 is that there is no whole number that can be multiplied by 3 to give 10. If you do long division for this problem, there will be a **remainder** that is not zero:

$$\begin{array}{r} 3 \\ 3\overline{)10} \\ \underline{9} \\ 1 \end{array} \leftarrow \text{Remainder}$$

What does this remainder mean? In this example, the remainder 1 is still to be divided by 3, so it can be written as $\frac{1}{3}$. Such an expression is called a **fraction** or a **rational number**.* The word *fraction* comes from a Latin word meaning "to break." A fraction involves two numbers: one "upstairs," called the **numerator**, and one

*Rational numbers are discussed in Section 2.6.

"downstairs," called the **denominator**. The denominator tells us into how many parts the whole has been divided, and the numerator tells us how many of those parts we have.

Fraction

A **fraction** is a number that is the quotient of a whole number divided by a counting number. A fraction is usually written as

$$\frac{\text{NUMERATOR}}{\text{DENOMINATOR}}$$

← A whole number
← Divided by
← A counting number (so that division by zero is excluded)

A fraction is called

- A **proper fraction** if the numerator is less than the denominator.
- An **improper fraction** if the numerator is greater than the denominator.
- A **whole number** if the denominator divides evenly into the numerator—that is, with remainder zero.

EXAMPLE 2 Classifying fractions

Classify each example as a proper fraction, an improper fraction, or a whole number

a. $\frac{6}{7}$ **b.** $\frac{6}{8}$ **c.** $\frac{7}{6}$ **d.** $\frac{0}{4}$ **e.** $\frac{6}{3}$ **f.** $\frac{4}{0}$

Solution

a. $\frac{6}{7}$ Proper **b.** $\frac{6}{8}$ Proper **c.** $\frac{7}{6}$ Improper **d.** $\frac{0}{4}$ Whole number

e. $\frac{6}{3}$ Whole number **f.** $\frac{4}{0}$ None of these (Don't forget, you can't divide by zero.)

Improper fractions that are not whole numbers can also be written in a form called **mixed numbers** by carrying out the division and leaving the remainder as a fraction. A mixed number thus has two parts: a counting number part and a proper fraction part.

EXAMPLE 3 Writing an improper fraction as a mixed number

Write $\frac{23}{5}$ as a mixed number.

Solution

$$\begin{array}{r} 4 \\ 5\overline{)23} \\ \underline{20} \\ 3 \end{array}$$

← Counting number part

← Remainder means 3 to be divided by 5, or $\frac{3}{5}$.

$\frac{23}{5} = 4 + \frac{3}{5}$ or $4\frac{3}{5}$.

EXAMPLE 4 Writing a mixed number as an improper fraction

Write $4\frac{3}{5}$ as an improper fraction.

Solution Reverse the procedure in Example 3. The remainder is 3, the quotient is 4, and the divisor is 5. Thus,

Whole number Numerator

↓ ↓

$4 \times 5 + 3 = 23$

↑

This is the divisor (the denominator of the fraction).

$$4\frac{3}{5} = \frac{23}{5} \qquad \leftarrow 23 = 4 \times 5 + 3$$

↑

This is the divisor (or the denominator of the fraction).

Decimal Fractions

We find certain fractions to be of special interest in our study of arithmetic. These are fractions with denominators that are powers of 10 (10, 100, 1,000, 10,000, . . .).

Tenths: $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}, \frac{11}{10}, \ldots$

Hundredths: $\frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \frac{4}{100}, \frac{5}{100}, \frac{6}{100}, \ldots$

Thousandths: $\frac{1}{1{,}000}, \frac{2}{1{,}000}, \frac{3}{1{,}000}, \frac{4}{1{,}000}, \frac{5}{1{,}000}, \ldots$

Each of these fractions can be written in **decimal form**:

$$\frac{1}{10} = 0.1 \qquad \frac{1}{100} = 0.01 \qquad \frac{1}{1{,}000} = 0.001$$

The word *fraction* is used with both forms. If we want to be specific, fractions written in the form

$$\frac{1}{10} \qquad \frac{1}{2} \qquad \frac{2}{5}$$

are called **common fractions**, and fractions written in the form

$$0.1 \quad 0.5 \quad 0.6$$

are called **decimal fractions**. When we simply use the word *fraction*, we are referring to a common fraction.

We will use division to show the relationship between fractions written as common fractions or as decimal fractions, but first let's look again at the positional notation of our number system:

If you do not already know these place-value (or column) names, you should learn them now.

Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	**Thousands**	**Hundreds**	**Tens**	Units	← Decimal point	Tenths, $\frac{1}{10}$	Hundredths, $\frac{1}{100}$	Thousandths, $\frac{1}{1{,}000}$	Ten-thousandths, $\frac{1}{10{,}000}$	Hundred-thousandths, $\frac{1}{100{,}000}$	Millionths, $\frac{1}{1{,}000{,}000}$
1,	2	3	4,	5	6	7,	8	9	1,	2	3	4	.	8	9	0	1	2	3

Every whole number can be written in decimal form:

$$\begin{aligned}
0 &= 0. = 0.0 = 0.00 = 0.000 = 0.0000 = 0.00000 = \cdots \\
1 &= 1. = 1.0 = 1.00 = 1.000 = 1.0000 = \cdots \\
2 &= 2. = 2.0 = 2.00 = 2.000 = \cdots \\
3 &= 3. = 3.0 = 3.00 = \cdots \\
4 &= 4. = 4.0 = \cdots \\
5 &= 5. = \cdots \\
&\vdots
\end{aligned}$$

A period, called a **decimal point** in the decimal system, is used to separate the fractional part from the whole part. Sometimes zeros are placed after the decimal point or after the last digit to the right of the decimal point, as in

$$3.21 = 3.210 = 3.2100 = 3.21000 = 3.21\underbrace{0000\ldots}$$

These are called **trailing zeros**.

Now let's carry out division, bringing the decimal point straight up from the dividend to the quotient:

$$\begin{array}{r} 0.1 \\ \uparrow \\ 10\overline{)1.0} \\ \underline{1\,0} \\ 0 \end{array}$$

← *Decimal point is carried straight up from dividend to quotient.*

Fractions to Decimals

EXAMPLE 5 Changing to decimal fractions by division

Write the given fractions as decimals by performing long division.

a. $\frac{3}{10}$ **b.** $\frac{7}{100}$ **c.** $\frac{137}{1,000}$

Solution

a. $\frac{3}{10}$; $\begin{array}{r} 0.3 \\ 10\overline{)3.0} \\ \underline{3\,0} \\ 0 \end{array}$ **b.** $\frac{7}{100}$; $\begin{array}{r} 0.07 \\ 100\overline{)7.00} \\ \underline{7\,00} \\ 0 \end{array}$ **c.** $\frac{137}{1,000}$; $\begin{array}{r} 0.137 \\ 1,000\overline{)137.000} \\ \underline{100\,0} \\ 37\,00 \\ \underline{30\,00} \\ 7\,000 \\ \underline{7\,000} \\ 0 \end{array}$

Place as many trailing zeros as are necessary.

Do you see a pattern in Example 5? Look at the next example.

EXAMPLE 6 Changing to decimal fractions by inspection

Write the given fractions in decimal form without doing any calculations.

a. $\frac{6}{10}$ **b.** $\frac{6}{100}$ **c.** $\frac{243}{1,000}$ **d.** $\frac{47}{10}$ **e.** $\frac{47}{100}$ **f.** $\frac{47}{1,000}$

Solution

a. $\frac{6}{10} = 0.6$ Tenths indicate one decimal place.

b. $\frac{6}{100} = 0.06$ Hundredths indicate two decimal places.

c. $\frac{243}{1,000} = 0.243$ Thousandths indicate three decimal places.

d. $\frac{47}{10} = 4.7$ **e.** $\frac{47}{100} = 0.47$ **f.** $\frac{47}{1,000} = 0.047$

You can see that fractions with denominators of 10, 100, 1,000, and so on are what we've called *decimal fractions* or simply **decimals**. On the other hand, *any fraction*, even one whose denominator is not a power of 10, can be written in decimal form by dividing.

EXAMPLE 7 Changing common fractions into decimals by using division

Write the given fractions in decimal form.

a. $\frac{3}{8}$ **b.** $\frac{7}{5}$ **c.** $2\frac{3}{4}$

Solution

a. $\frac{3}{8}$;

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{2\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Remember, the decimal point is moved straight up from the dividend to the quotient. Otherwise, carry out the division in the usual fashion.

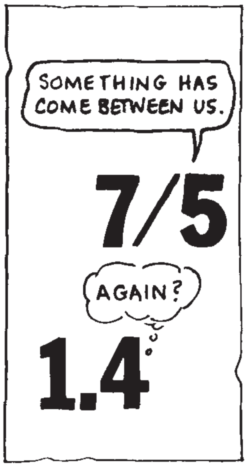
You may keep adding trailing zeros here as long as you wish.

Thus, $\frac{3}{8} = 0.375$. You can obtain this result using a calculator. If a zero remainder is obtained, the decimal is called a **terminating decimal**.

b. $\frac{7}{5}$;

$$\begin{array}{r} 1.4 \\ 5\overline{)7.0} \\ \underline{5} \\ 2.0 \\ \underline{2.0} \\ 0 \end{array}$$

Thus, $\frac{7}{5} = 1.4$.



Cartoon courtesy of Patrick J. Boyle.

c. $2\frac{3}{4}$; Since $2\frac{3}{4} = \frac{11}{4}$, we divide 4 into 11:

$$\begin{array}{r} 2.75 \\ 4\overline{)11.00} \\ \underline{8} \\ 3\,0 \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Thus, $2\frac{3}{4} = 2.75$.

An alternative method is to notice that

$$2\frac{3}{4} = 2 + \frac{3}{4}$$

Divide 4 into 3:

$$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \end{array}$$

and then add 0.75 to 2.

We noted in part **a** of Example 7 that you can append as many trailing zeros after the 3.0 as you wish. For some fractions, you could continue to append zeros forever and never complete the division. Such fractions are called **repeating decimals**.

EXAMPLE 8 Changing common fractions (repeating decimals)

Write $\frac{2}{3}$ in decimal form.

Solution

$$\begin{array}{r} 0.666 \\ 3\overline{)2.000} \\ \underline{1\,8} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

← The same remainder keeps coming up, so the process is never finished.

CAUTION When you write repeating decimals, be careful to include three dots or the overbar.

This is a *repeating decimal*, as is indicated by three trailing dots or by a bar over the repeating digits:

$$\frac{2}{3} = 0.666\ldots \quad \text{or} \quad \frac{2}{3} = 0.\overline{6}$$

You cannot correctly write

$$\tfrac{2}{3} = 0.6666 \quad \text{or} \quad \tfrac{2}{3} = 0.6667$$

These are wrong!

because $3 \times 0.6666 = 1.9998$ and $3 \times 0.6667 = 2.0001$. You can correctly write

$$\tfrac{2}{3} \approx 0.6666 \quad \text{or} \quad \tfrac{2}{3} = 0.\overline{6}$$

The symbol $\approx$ means **approximately equal to**.

All fractions have a decimal form that either terminates (as in Example 7) or repeats (as in Example 8). Any number of digits may repeat; consider Example 9. Calculators do not write repeating decimals, but finding $\frac{2}{3}$ on your calculator can be instructive.

CAUTION

Press: $\boxed{2}\ \boxed{\div}\ \boxed{3}\ \boxed{=}$

The display may vary. If you see 0.666666667, then your calculator *rounds* the last decimal place, whereas if you see 0.666666666, then your calculator *truncates* (cuts off) at the last decimal place. You should remember whether your calculator rounds or truncates. You should also count the number of decimal places shown in your calculator's display. The number of places shown by this example indicates the accuracy of your calculator. The one shown here is accurate to eight decimal places. (Note that the ninth decimal place is not accurate because of possible rounding.)

Estimating Fractions

EXAMPLE 9 Changing a mixed fraction to a decimal fraction

Write $3\frac{5}{11}$ in decimal form.

Solution First, write $\frac{5}{11}$ as a decimal:

$$\begin{array}{r} 0.45\ldots \\ 11\overline{)5.00} \\ \underline{4\ 4} \\ 60 \\ \underline{55} \\ 5 \end{array} \leftarrow \text{Repeats}$$

By calculator: $\boxed{3}\ \boxed{+}\ \boxed{5}\ \boxed{\div}\ \boxed{11}\ \boxed{=}$

Display: 3.4545454545

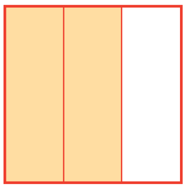
Remember, a calculator does not show beyond its display; for this one, you must notice that it is a repeating decimal when you write $3.\overline{45}$.

Thus, $3\frac{5}{11} = 3 + \frac{5}{11} = 3 + 0.\overline{45} = 3.\overline{45}$.

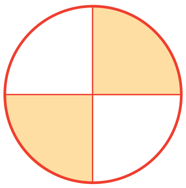
EXAMPLE 10 Estimating parts using common fractions

Estimate the size of each shaded portion as a common fraction.

a.



b.



Solution

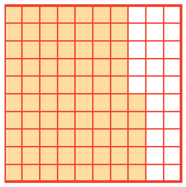
a. The square is divided into three parts, and two of the three parts are shaded, so we estimate that $\frac{2}{3}$ of the square is shaded.

b. The circle is divided into four parts, and two of the four parts are shaded, so we estimate that $\frac{2}{4}$ or $\frac{1}{2}$ of the circle is shaded.

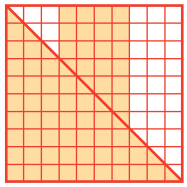
EXAMPLE 11 Estimating parts using decimal fractions

Estimate the size of each shaded portion as a decimal.

a.



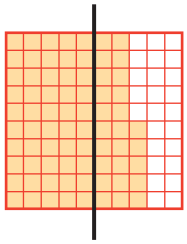
b.



If you resort to counting squares, you are not estimating.

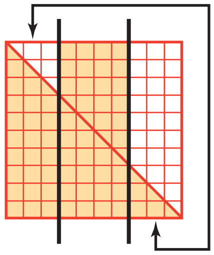
Solution

a. Divide the square in half:



It looks as if half the remaining half is shaded, so we estimate that 75 of the 100 squares are shaded. The shaded portion is 0.75.

b. Divide the square into thirds:



It appears that the number of squares not shaded in the first third is about the same as the number of small squares shaded in the last third, so we estimate the size of the shaded squares to be $\frac{2}{3}$ or $0.\overline{6}$.

EXAMPLE 12 Application to a stock purchase

Several years ago, you purchased 400 shares of Disney stock selling at $29\frac{5}{8}$ per share. What is the total price you paid for this stock?

Solution First convert the stock price to dollars and cents by changing the mixed number to dollars and cents; then multiply by the number of shares.

$$
\begin{aligned}
29\tfrac{5}{8} &= 29 + \tfrac{5}{8} \\
&= 29 + 0.625 \\
&= 29.625
\end{aligned}
$$

You might wish to do this by calculator.

5 ÷ 8 = 0.625

Do not round until you are ready to state your answer.

Thus, the total price is $400 \times \$29.625 = \$11{,}850$.

Problem Set 1.3

The POWER of Math

1. **IN YOUR OWN WORDS** Explain why division by zero is not permitted.
2. **IN YOUR OWN WORDS** Distinguish between a common fraction and a decimal fraction.

LEVEL 1 Essential Ideas

3. **IN YOUR OWN WORDS** What is a fraction?
4. **IN YOUR OWN WORDS** Use a calculator to carry out the following sets of calculations.
   a. [1] [÷] [1] [=]
   b. [1] [÷] [.] [1] [=]

c. $\boxed{1}\ \boxed{\div}\ \boxed{.}\ \boxed{0}\ \boxed{1}\ \boxed{=}$

d. $\boxed{1}\ \boxed{\div}\ \boxed{.}\ \boxed{0}\ \boxed{0}\ \boxed{1}\ \boxed{=}$

e. $\boxed{1}\ \boxed{\div}\ \boxed{.}\ \boxed{0}\ \boxed{0}\ \boxed{0}\ \boxed{1}\ \boxed{=}$

Describe the pattern of problems and answers. Does this lead you to any conclusion?

5. Give the names of the positional columns from trillions to millionths.

LEVEL 1 Right or Wrong?

Explain what is wrong, if anything, with the statements in Problems 6-13. Explain your reasoning.

6. $\frac{0}{2} = 0$

7. $\frac{10}{0} = 0$

8. $0.05 = 0.050$

9. 0.75 is a fraction

10. $\frac{1}{3} = 0.333333333$

11. $0.\overline{6} = 0.666666666$

12. $238 \div 8.5 = 28$

13. $238 \div 8.51 = 27.9670$

LEVEL 2 Drill and Practice

Classify each fraction as proper, improper, or a whole number, and then show the keystrokes required to enter each of the fractions in Problems 14-15 into a calculator.

14. a. $\frac{3}{2}$
b. $\frac{15}{5}$

15. a. $\frac{8}{13}$
b. $\frac{17}{21}$

Show the keystrokes required to enter each of the mixed numbers in Problems 16-17 into a calculator.

16. a. $14\frac{3}{4}$
b. $3\frac{2}{7}$

17. a. $4\frac{3}{7}$
b. $2\frac{1}{2}$

Write each of the improper fractions in Problems 18-23 as a mixed number or a whole number.

18. a. $\frac{3}{2}$ **b.** $\frac{4}{3}$ **c.** $\frac{5}{4}$ **d.** $\frac{19}{2}$

19. a. $\frac{16}{3}$ **b.** $\frac{25}{4}$ **c.** $\frac{141}{10}$ **d.** $\frac{163}{10}$

20. a. $\frac{1,681}{10}$ **b.** $\frac{1,493}{10}$ **c.** $\frac{833}{100}$ **d.** $\frac{1,457}{100}$

21. a. $\frac{27}{16}$ **b.** $\frac{177}{10}$ **c.** $\frac{33}{16}$ **d.** $\frac{27}{7}$

22. a. $\frac{41}{12}$ **b.** $\frac{61}{5}$ **c.** $\frac{118}{15}$ **d.** $\frac{89}{21}$

23. a. $\frac{83}{5}$ **b.** $\frac{42}{3}$ **c.** $\frac{18}{3}$ **d.** $\frac{125}{4}$

Write each of the mixed numbers in Problems 24-35 as an improper fraction.

24. a. $2\frac{1}{2}$ **b.** $1\frac{2}{3}$

25. a. $1\frac{3}{4}$ **b.** $3\frac{3}{5}$

26. a. $4\frac{3}{8}$ **b.** $5\frac{1}{4}$

27. a. $3\frac{2}{3}$ **b.** $5\frac{1}{5}$

28. a. $6\frac{1}{2}$ **b.** $3\frac{2}{5}$

29. a. $1\frac{3}{10}$ **b.** $4\frac{2}{5}$

30. a. $2\frac{2}{5}$ **b.** $3\frac{7}{10}$

31. a. $17\frac{2}{3}$ **b.** $12\frac{4}{5}$

32. a. $11\frac{3}{8}$ **b.** $2\frac{9}{10}$

33. a. $1\frac{14}{15}$ **b.** $2\frac{13}{17}$

34. a. $19\frac{3}{5}$ **b.** $17\frac{1}{8}$

35. a. $18\frac{7}{8}$ **b.** $3\frac{11}{12}$

Change the fractions and mixed numbers in Problems 36-47 into decimal fractions.

36. a. $\frac{1}{8}$ **b.** $\frac{3}{8}$

37. a. $\frac{5}{6}$ **b.** $\frac{7}{6}$

38. a. $\frac{3}{5}$ **b.** $\frac{7}{8}$

39. a. $2\frac{1}{2}$ **b.** $5\frac{1}{3}$

40. a. $\frac{3}{7}$ **b.** $3\frac{1}{6}$

41. a. $6\frac{1}{12}$ **b.** $6\frac{2}{3}$

42. a. $\frac{5}{9}$ **b.** $\frac{7}{9}$

43. a. $7\frac{5}{6}$ **b.** $4\frac{1}{15}$

44. a. $3\frac{1}{30}$ **b.** $\frac{2}{7}$

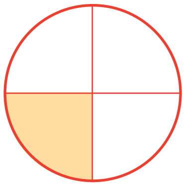
45. a. $2\frac{2}{3}$ **b.** $4\frac{1}{6}$

46. a. $\frac{7}{15}$ **b.** $\frac{8}{9}$

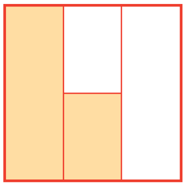
47. a. $4\frac{3}{8}$ **b.** $3\frac{1}{9}$

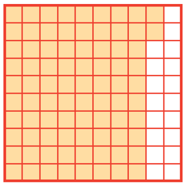
Estimate as a common fraction the size of the shaded portion of the regions in Problems 48-49.

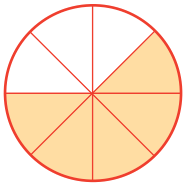
48. a.



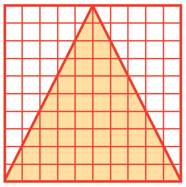
b.

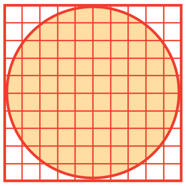


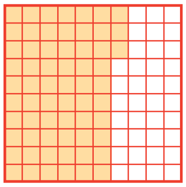
49. a.


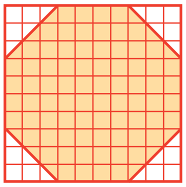
b.


Estimate as a decimal fraction the shaded portions for each of the regions in Problems 50-51.

50. a.


b.


51. a.


b.


In Problems 52-54, *use a calculator to write each fraction in decimal form. Give the exact decimal value, if possible. If you can't give the exact value, indicate that the answer you are giving is approximate.*

52. a. $\frac{1}{11}$ **b.** $\frac{1}{12}$

53. a. $\frac{7}{15}$ **b.** $\frac{7}{22}$

54. a. $\frac{5}{19}$

b. $\frac{3}{17}$

LEVEL 2 Applications

Stock prices used to be quoted using eighths of a dollar. Find the total price of each transaction in Problems 55-60.

55. 100 shares of Citicorp at $52\frac{1}{2}$

56. 100 shares of Liz Claiborne at $18\frac{3}{4}$

57. 200 shares of Apple Computer at $42\frac{1}{4}$

58. 300 shares of National Semi at $126\frac{1}{2}$

59. 100 shares of Delta at $63\frac{1}{8}$

60. 100 shares of Genentech at $60\frac{5}{8}$

1.4 Rounding and Estimation

The POWER of Math

"Calling my family in Mexico is costing me a fortune," Miguel moaned. "My phone bill was an extra $40 last month alone . . . and that didn't even include all my texting!"

"You should really get an international phone card," responded Sally. "Be careful though—some have one-minute, two-minute, or even three-minute rounding. They really try to rip you off."

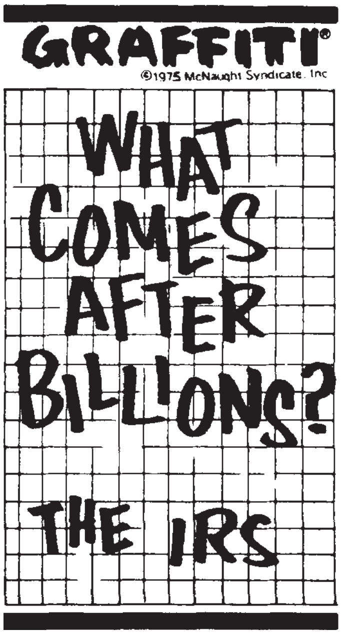
Miguel shrugged. "Three-minute rounding? What the heck's that?"

"If you talk for even one second over three minutes, they will round up and bill you for six," Sally explained. "Consider yourself warned!"

In this section, we will consider the process of rounding.

Understanding Large Numbers

Very large or very small numbers are difficult to comprehend. Most of us are accustomed to hearing about millions, billions, and even trillions, but do we really understand the magnitude of these numbers?



If you were to count one number per second, nonstop, it would take about 278 hours, or approximately $11\frac{1}{2}$ days, to count to a million. Not a million days have elapsed since the birth of Christ (a million days is about 2,700 years). A large book of about 700 pages contains about a million letters. A million bottle caps placed in a single line would stretch about 17 miles.

But the age in which we live has been called the age of billions. How large is a billion? How long would it take you to count to a billion? Go ahead—make a guess.

To get some idea about how large a billion is, let's compare it to some familiar units:

- If you gave away $1,000 *per day*, it would take you more than 2,700 *years* to give away a billion dollars.
- A stack of a billion $1 bills would be more than 59 miles high.
- At 8% interest, a billion dollars would earn you $219,178.08 interest *per day*!
- A billion seconds ago, *Star Wars: Episode V* and *Pac-Man* were released.
- A billion minutes ago was about a.d. 110.
- A billion hours ago, people had not yet appeared on the earth.

Rounding Numbers

To carry out the calculations necessary to make the preceding statements, we need two ideas. The first is rounding (discussed in this section), and the second is scientific notation (discussed in the next section). Rounding, estimation, and scientific notation can help to increase the clarity of our understanding of numbers. For example, a budget of $252,892,988.18 for 99,200 students can be approximated as a $250 million budget for 100,000 students. A measurement of $19\frac{15}{16}$ inches might be recorded as 20 inches. You are often required to round decimals (for example, when you work with money). Estimates for the purpose of clarity are almost always found by rounding.

What do we mean by **rounding**? The following procedure should help to explain.

Rounding a Decimal

To round a number in decimal form:

Step 1 **Locate the rounding digit.** This is identified by the column name. (These are listed on page 22.) Also, we sometimes refer to *two-place accuracy* and mean that the decimal should be accurate to two digits after the decimal point, and similarly for *three-place accuracy, four-place accuracy, ...*

Step 2 **Determine the rounding place digit.**
- It stays the same if the first digit to its right is a 0, 1, 2, 3, or 4.
- It increases by 1 if the digit to the right is a 5, 6, 7, 8, or 9. (If the rounding place digit is a 9 and 1 is added, there will be a carry in the usual fashion.)

Step 3 **Change digits.**
- All digits to the left of the rounding digit remain the same (unless there is a carry).
- All digits to the right of the rounding digit are changed to zeros.

Step 4 **Drop zeros.**
- If the rounding place digit is to the left of the decimal point, drop all trailing zeros.
- If the rounding place digit is to the right of the decimal point, drop all trailing zeros to the right of the rounding place digit.

EXAMPLE 1 Rounding

Round 46.8217 to the nearest hundredth.

Solution

Step 1. The rounding place digit is in the hundredths column.

Step 2. Rounding place digit

4 6 . 8 | 2 | 1 7

The digit to the right of the rounding digit is 1, so the rounding digit stays the same.

Step 3. These digits stay the same.

Step 4. These digits are changed into 0s; drop trailing zeros.

The rounded number is 46.82.

EXAMPLE 2 Rounding with a carry

Round 13.6992 to the nearest hundredth.

Solution

Step 1. The rounding place digit is in the hundredths column.

Step 2. Rounding place digit

1 3 . 6 | 9 | 9 2

The digit to the right of the rounding digit is 9, so the rounding digit is increased by 1.

Step 3. These digits stay the same (except for a carry).

Step 4. These digits are changed into 0s; drop trailing zeros.

The rounded number is 13.70. Notice that the zero that appears as the rounding place digit is *not* deleted.

EXAMPLE 3 Rounding; drop trailing zeros

Round 72,416.921 to the nearest hundred.

Solution

Step 1. The rounding place digit is in the hundreds column.

Step 2. Rounding place digit

7 2, | 4 | 1 6 . 9 2 1

The digit to the right of the rounding digit is 1, so the rounding digit stays the same.

Step 3. These digits stay the same.

Step 4. These digits are changed into 0s; drop trailing zeros.

$72{,}400.\underbrace{000}_{\text{Delete these trailing zeros.}}$

STOP Note the agreement about rounding used in this book.

The rounded number is 72,400.

In this book, we will round answers dealing with money to the nearest cent, and you should do the same in your work. Otherwise, you should not round any of your answers unless you are instructed to do so. This means, for example, that if you are converting fractions to decimals, you must include all decimal places until the decimal terminates or repeats (and you show this with an overbar). If you are using a calculator, you will need to interpret the display as representing either a terminating or a repeating decimal, if possible.

Estimation

We first introduced estimation in Section 1.2. Remember, estimation is obtaining a reasonably accurate guess. One of the most important estimation tools is rounding. Throughout this book, we will provide multiple-choice estimation questions so that you can practice and strengthen your estimating skills.

EXAMPLE 4 Estimation by rounding

Divide \$858.25 into three shares. Which of the following is the best estimate of one share?

A. \$90 B. \$300 C. \$900

Solution Mentally round \$858.25 to the nearest hundred dollars (\$900); then divide by 3 to obtain \$300. Thus, the best estimate is B.

EXAMPLE 5 Estimating by rounding

A theater has 53 rows with 39 seats in each row. Which of the following is the best estimate of the number of seats in the theater?

A. 100 B. 1,000 C. 2,000

Solution Mentally round to the nearest ten: 50 rows of 40 seats each would give $50 \times 40 = 2,000$ seats. Thus, the best estimate is C.

Problem Set 1.4

The POWER of Math

1. **IN YOUR OWN WORDS** Describe the process of rounding.
2. Suppose you make a call to Mexico once a week and talk for 21 minutes and 5 seconds each time. You have a choice of the following \$20.00 phone cards:

Card	Fee	Rounding	Price
HELLO	\$0.99/wk	4 min	1.8¢/min
TALK	\$0.69/wk	2 min	2.4¢/min
ANYTIME	none	1 sec	5.5¢/min

Which card should you choose?

LEVEL 1 Essential Ideas

STOP Don't forget to look at all of these essential ideas throughout the book.

3. **IN YOUR OWN WORDS** What is a rounding place digit?
4. **IN YOUR OWN WORDS** Why is estimation an important technique in mathematics?

LEVEL 1 Right or Wrong?

Explain what is wrong, if anything, with the statements in Problems 5-10. *Explain your reasoning.*

5. 30.05 rounded to the nearest tenth is 30.

6. 625.97555 rounded to the nearest hundredth is 600.

7. 3,684,999 rounded to the nearest ten thousand is 3,690,000.

8. 12,456.9099 rounded to the nearest tenth is 12,456.9000.

9. If a jar contains 23 jelly beans, then 12 similar jars will contain about 250 jelly beans.

10. If a box contains 144 pencils, then obtaining 1,000 pencils will require about 7 boxes.

LEVEL 2 Drill and Practice

Round the numbers in Problems 11-32 *to the given degree of accuracy.*

11. 2.312; nearest tenth

12. 14.836; nearest tenth

13. 6,287.4513; nearest hundredth

14. 342.355; nearest hundredth

15. 5.291; one decimal place

16. 5.291; two decimal places

17. 6,287.4513; nearest hundred

18. 6,287.4513; nearest thousand

19. 12.8197; two decimal places

20. 813.055; two decimal places

21. 4.81792; nearest thousandth

22. 1.396; nearest unit

23. 4.8199; nearest whole number

24. 48.5003; nearest whole number

25. $12.993; nearest cent

26. $6.4312; nearest cent

27. $14.998; nearest cent

28. $6.9741; nearest cent

29. 694.3814; nearest ten

30. 861.43; nearest hundred

31. $86,125; nearest thousand dollars

32. $125,500; nearest thousand dollars

Round the calculator display shown in Problems 33-38.

```
2/3
       .6666666667
2/17
       .1176470588
7/51
        .137254902
```

```
1/3
       .3333333333
5/11
       .4545454545
19/53
        .358490566
```

33. 2/3; three-place accuracy

34. 1/3; four-place accuracy

35. 2/17; three-place accuracy

36. 5/11; four-place accuracy

37. 7/51; three-place accuracy

38. 19/53; four-place accuracy

LEVEL 2 Applications

*A baseball player's batting average is found by dividing the number of hits by the number of times at bat. This number is then rounded to the nearest thousandth. Find each player's batting average in Problems*39-44.

39. Cole Becker was at bat 6 times with 2 hits.

40. Hannah Becker was at bat 20 times with 7 hits.

41. Theron Sovndal had 5 hits in 12 times at bat.

42. Jeff Tredway had 98 hits in 306 times at bat.

43. Hal Morris had 152 hits in 478 times at bat.

44. Tony Gwynn had 168 hits in 530 times at bat.

Estimate answers for Problems 45-50 *choosing the most reasonable answer. You should not do any pencil-and-paper or calculator arithmetic for these problems.*

45. The distance to school is 4.82 miles, and you must make the trip to school and back five days a week. Estimate how far you drive each week.

A. 9.64 miles

B. 50 miles

C. 482 miles

46. The length of a lot is 279 ft, and you must order fencing material that requires six times the length of the lot. Estimate the amount of material you should order.

A. 1,600 ft B. 50 ft C. 10,000 ft

47. If a person's annual salary is $35,000, estimate the monthly salary.

A. $500 B. $1,000 C. $3,000

48. If 500 shares of a stock cost $19,087.50, estimate the value of each share of stock.

A. $40 B. $400 C. $4,000

49. If you must pay back $1,000 in 12 monthly installments, estimate the amount of each payment.

A. $25 B. $100 C. $12,000

50. If an estate of $22,000 is to be divided equally among three children, estimate each child's share.

A. $7,000 B. $733.33 C. $65

First give an estimate before answering each question in Problems 51-58.

51. If a person's annual salary is \$15,000, what is the monthly salary?

52. If a person's annual tax is \$512, how much is that tax per month?

53. If the sale of 150 shares of PERTEC stock grossed \$1,818.75, how much was each share worth?

54. If an estate of \$378,459 is to be divided equally among three children, what is each child's share?

55. If you must pay back \$850 in 12 monthly payments, what is the amount of each payment?

56. If you must pay back \$1,000 in 12 monthly payments, what is the amount of each payment?

57. A businesswoman bought a copy machine for her office. If it cost \$674 and the useful life is six years, what is the cost per year?

58. A businessman bought a fax machine for his office. If it cost \$890 and the useful life is seven years, what is the cost per year?

59. IN YOUR OWN WORDS Describe the size of a million.

60. IN YOUR OWN WORDS Describe the size of a billion.

1.5 Exponents and Prime Factorization

The POWER of Math

"I read today that our national debt is over \$14 trillion," Anita declared with dismay. "And it grows by millions every day! We'll never be able to pay it off."

"You know, it's not as bad as you think," replied Rita. "Our debt may appear overwhelming in absolute dollars, but try to look at it as a percentage of gross national product instead."

"Well you're right about one thing—the state of our economy is pretty gross."

In this section, we will devise a way to handle big numbers, such as those dealing with national debt.

We often encounter numbers that are made by repeated multiplication of the same numbers. For example,

$$10 \times 10 \times 10 \qquad 6 \times 6 \times 6 \times 6 \times 6$$
$$15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15$$

Exponents

These numbers can be written more simply by inventing a new notation:

$$10^3 = \underbrace{10 \times 10 \times 10}_{3\ \text{factors}}$$

$$6^5 = \underbrace{6 \times 6 \times 6 \times 6 \times 6}_{5\ \text{factors}}$$

$$15^{13} = \underbrace{15 \times 15 \times \cdots \times 15}_{13\ \text{factors}}$$

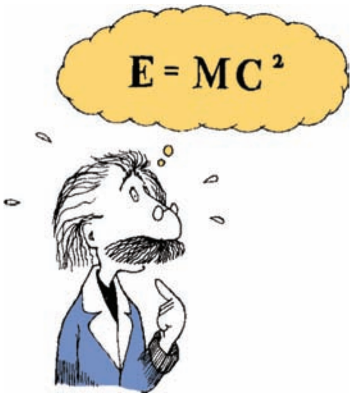
> **Historical Note**
>
> For historical, geometrical reasons, special terminology is used only with exponents of 2 or 3. The area of a square with side s is s^2, so the exponent 2 is pronounced "**squared.**" In the volume of a cube, the exponent is 3 and is pronounced "**cubed.**"

We call this **power** or **exponential notation**. The number that is multiplied as a repeated factor is called the **base**, and the number of times the base is used as a factor is called the **exponent**. To use your calculator for exponents, locate the $\boxed{y^x}$, $\boxed{x^y}$, or $\boxed{\wedge}$ keys. For example, to find the value of 6^5,

$$\boxed{6}\ \boxed{\wedge}\ \boxed{5}\ \boxed{=}$$

You should see 7,776 displayed.

EXAMPLE 1 Definition of exponent



Tell what each of the given expressions means, and then expand (multiply out). Use your calculator where appropriate.

a. 6^2 **b.** 10^3 **c.** 7^5 **d.** 2^{15}

Solution

a. $6^2 = 6 \times 6$ or 36

The base is 6; the exponent is 2; this is pronounced "six squared."

b. $10^3 = 10 \times 10 \times 10 = 1{,}000$

The base is 10; the exponent is 3; this is pronounced "ten cubed."

c. $7^5 = 7 \times 7 \times 7 \times 7 \times 7 = 16{,}807$ (by calculator)

The base is 7; the exponent is 5; this is pronounced "seven to the fifth power."

d. $2^{15} = \underbrace{2 \times 2 \times 2 \times \cdots \times 2 \times 2}_{\text{15 factors}} = 32{,}768$

We now use this notation to observe a pattern for **powers of 10**:

$$10^1 = 10$$
$$10^2 = 10 \times 10 = 100$$
$$10^3 = 10 \times 10 \times 10 = 1{,}000$$
$$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$$

Do you see a relationship between the exponent and the number?

$$10^{\underset{\text{Exponent is 5.}}{5}} = 1\underbrace{00{,}000}_{\text{5 zeros}}$$ *Notice that the exponent and the number of zeros are the same.*

Could you write 10^{12} without actually multiplying?*

Scientific Notation

There is a similar pattern for multiplication of any number by a power of 10. Consider the following examples, and notice what happens to the decimal point:

$9.42 \times 10^1 = 94.2$ *We find these answers by direct multiplication.*

$9.42 \times 10^2 = 942.$

$9.42 \times 10^3 = 9{,}420.$

$9.42 \times 10^4 = 94{,}200.$

*Answer: 1,000,000,000,000.

Do you see a pattern? If we multiply 9.42×10^5, how many places to the right will the decimal point be moved?

$$9.42 \times 10^5 = 9\;42{,}000$$

5 places →

This answer is found by observing the pattern, not by directly multiplying.

Using this pattern, can you multiply the following *without direct calculation?**

$$9.42 \times 10^{12}$$

We will investigate one final pattern of 10s, this time looking at smaller values in this pattern:

$$\begin{aligned}
&\vdots\\
100{,}000 &= 10^5\\
10{,}000 &= 10^4\\
1{,}000 &= 10^3\\
100 &= 10^2\\
10 &= 10^1
\end{aligned}$$

Continuing with the same pattern, we have

1	$= 10^0$	We interpret the zero exponent as "decimal point moves 0 places."
0.1	$= 10^{-1}$	We will define negative numbers in Chapter 2.
0.01	$= 10^{-2}$	For now, we use the symbols $-1, -2, -3, \ldots$ as exponents to show position of decimal point.
0.001	$= 10^{-3}$	
0.0001	$= 10^{-4}$	
0.00001	$= 10^{-5}$	
$\vdots$		

When we multiply some number by a power of 10, a pattern emerges:

$9.42 \times 10^2 = 942.$	
$9.42 \times 10^1 = 94.2$	
$9.42 \times 10^0 = 9.42$	Decimal moves 0 places.
$9.42 \times 10^{-1} = 0.942$	These answers are found by direct multiplication: $9.42 \times 0.1 = 0.942$.
$9.42 \times 10^{-2} = 0.0942$	
$9.42 \times 10^{-3} = 0.00942$	

Do you see that the same pattern holds for multiplying by a negative exponent? Can you multiply 9.42×10^{-6} *without direct calculation?* The solution is as follows:

$$9.42 \times 10^{-6} = 0.\,00000\,9\,42$$

← Moved 6 places to the left.

These patterns lead to a useful way of writing large and small numbers, called **scientific notation**.

Scientific Notation

The **scientific notation** of a number is that number written as a power of 10 or as a decimal number between 1 and 10 times a power of 10.

*Answer: 9,420,000,000,000.

EXAMPLE 2 Writing numbers in scientific notation

Write the given numbers in scientific notation.

a. 123,600 **b.** 0.000035 **c.** 48,300 **d.** 0.0821 **e.** 1,000,000,000,000 **f.** 7.35

Solution

a. $123{,}600 = 1.\,236 \times 10^{?}$

$\uparrow$

Step 1. Fix the decimal point after the first nonzero digit.

Step 2. From this number, count the number of decimal places to restore the numberto its given form:

$$123{,}600 = 1.\,23600 \times 10^{?}$$

$\downarrow$ $\uparrow$

Move decimal point 5 places to the right $\rightarrow$

Step 3. The exponent is the same as the number of decimal places needed to restorescientific notation to the original given number:

$$123{,}600 = 1.236 \times 10^{5}$$

b. $0.000035 = 3.\,5 \times 10^{?}$

$\uparrow$

Step 1. Fix the decimal point.

Step 2. $0.000035 = 0\,00003.\,5 \times 10^{?}$

$\uparrow$ $\downarrow$

$\leftarrow$ 5 places to the left; this is -5.

Step 3. $0.000035 = 3.5 \times 10^{-5}$

c. $48{,}300 = 4.83 \times 10^{4}$

d. $0.0821 = 8.21 \times 10^{-2}$

e. $1{,}000{,}000{,}000{,}000 = 10^{12}$

f. $7.35 = 7.35 \times 10^{0}$ or just 7.35

When we are working with very large or very small numbers, it is customary to use scientific notation. For example, suppose we wish to expand 2^{63}. A calculator can help us with this calculation:

$\boxed{2}\ \boxed{\frown}\ \boxed{63}\ \boxed{=}$

The result is larger than can be handled with a calculator display, so your calculator will automatically output the answer in scientific notation, using one of the following formats:

9.223372037E18 9.223372037 18 9.223372037 $\times 10^{18}$

Do not confuse the exponent key $\boxed{y^x}$ with scientific notation $\boxed{EE}$; they are NOT the same.

If you wish to enter a v ery large (or small) number into a calculator , you can enter these numbers using the scientific notation button on most calculators. Use the k ey labeled $\boxed{EE}$, $\boxed{EXP}$, or $\boxed{SCI}$. Whenever we show the $\boxed{EE}$ key, we mean press the scientific notation key on your brand of calculator.

Number	*Scientific Notation*	*Calculator Input*	*Calculator Display*
468,000	4.68×10^{5}	$\boxed{4.68}\ \boxed{EE}\ \boxed{5}$	4.68 05 or 4.68 E5
93,000,000,000	9.3×10^{10}	$\boxed{9.3}\ \boxed{EE}\ \boxed{10}$	9.3 10 or 9.3 E10

EXAMPLE 3 Operations with scientific notation

If the federal budget is \$1.5 trillion (1,500,000,000,000), how much does it cost each individual, on average, if there are 240,000,000 people?

Solution We must divide the budget by the number of people. We will use scientific notation and a calculator:

$$1{,}500{,}000{,}000{,}000 = 1.5 \times 10^{12} \quad \text{and} \quad 240{,}000{,}000 = 2.4 \times 10^{8}$$

The desired calculation is $\dfrac{1.5 \times 10^{12}}{2.4 \times 10^{8}}$.

We carry out this calculation using the appropriate scientific notation keys:

$\boxed{1.5}$ $\boxed{\text{EE}}$ $\boxed{12}$ $\boxed{\div}$ $\boxed{2.4}$ $\boxed{\text{EE}}$ $\boxed{8}$ $\boxed{=}$

The answer is \$6,250 per person.

Extended Order of Operations

In Example 3, multiplications and exponents are mixed in the scientific notation, and sometimes we need to mix exponents and additions or subtractions. Since an exponent is an indicated multiplication, the proper procedure is first to simplify the exponent and then to carry out the multiplication. This leads to an **extended order-of-operations** agreement.

Order of Operations Extended

In doing arithmetic with mixed operations, we agree to proceed by following these steps:

Step 1 Parentheses first

Step 2 Perform any operations that involve raising to a power

Step 3 Multiplications and divisions, reading from left to right

Step 4 Additions and subtractions, reading from left to right

STOP This is essential for almost all that follows. You must make its application second nature.

EXAMPLE 4 Using the extended order of operations

Simplify $2^4 + 3(5 + 6)$.

Solution

$$\begin{aligned} 2^4 + 3(5+6) &= 2^4 + 3(11) && \text{Parentheses first} \\ &= 16 + 3(11) && \text{Powers next} \\ &= 16 + 33 && \text{Multiplications/divisions} \\ &= 49 && \text{Additions/subtractions} \end{aligned}$$

Factoring

When you multiply numbers, the numbers being multiplied are called **factors**. The process of taking a given number and writing it as the product of two or more other numbers is called **factoring**, with the result called a **factorization** of the given number.

EXAMPLE 5 Finding factors

Find the factors of the given numbers.

a. 1 **b.** 2 **c.** 3 **d.** 4 **e.** 5 **f.** 6 **g.** 7 **h.** 8

Solution

a. $1 = 1 \times 1$ The factor is 1.

b. $2 = 2 \times 1$ Factors: 1, 2

c. $3 = 3 \times 1$ Factors: 1, 3

d. 4 $= 4 \times 1$ or 2×2 Factors: 1, 2, 4
e. 5 $= 5 \times 1$ Factors: 1, 5
f. 6 $= 6 \times 1$ or 2×3 Factors: 1, 2, 3, 6
g. 7 $= 7 \times 1$ Factors: 1, 7
h. 8 $= 8 \times 1$ or 4×2 or $2 \times 2 \times 2$ Factors: 1, 2, 4, 8

Prime Numbers

Let's categorize the numbers listed in Example 5:

	PRIMES	*COMPOSITES*
Fewer than two factors:	*Exactly two factors:*	*More than two factors:*
1	2	4
	3	6
	5	8
	7	

A **prime number** is a natural number with exactly two distinct factors, and a **composite number** is a number with more than two factors. Will any number besides 1 have fewer than two factors? The primes smaller than 100 are listed in the box.

Primes Smaller Than 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

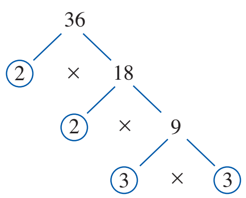
A **prime factorization** of a number is a factorization that consists exclusively of prime numbers.

EXAMPLE 6 Prime factorization of a number

Find the prime factorization of 36.

Solution

Step 1. From your knowledge of the basic multiplication facts, write any two numbers whose product is the given number. Circle any prime factor.



This process for finding a prime factorization is called a **factor tree**.

Step 2. Repeat the process for uncircled numbers.

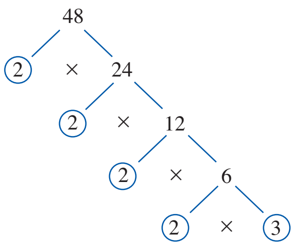
Step 3. When all the factors are circled, their product is the *prime factorization*.

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

EXAMPLE 7 Prime factorization of a number

Find the prime factorization of 48.

Solution

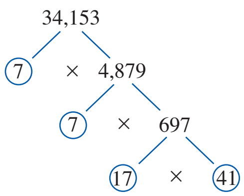


$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$$

If you cannot readily find the prime factors of a number, you should look at the list of prime factors from the smallest to the larger numbers, as illustrated in Example 8.

EXAMPLE 8 Prime factorization of a large number

Find the prime factorization of 34,153.



Solution First, try 2; you might notice that 34,153 is not an even number, so 2 is not a factor. Next, try 3 (the next prime after 2); 3 does not divide evenly into 34,153, so 3 is not a factor.

Next, try 5; it is also not a factor.

The next prime to try is 7 (by calculator or long division; calculator display is 4879). Now focus on 4,879. Previously tried numbers cannot be factors, but since 7 *was* a factor, it might be again: $4{,}879 = 7 \times 697$.

Continue this process: 697 is not divisible (evenly) by 7, so try 11 (doesn't divide evenly), 13 (doesn't divide evenly), and 17 (does). The remaining number, 41, is a prime, so the process is complete:

$$34{,}153 = 7^2 \times 17 \times 41$$

Problem Set 1.5

The POWER of Math

1. IN YOUR OWN WORDS When do you think you will ever be asked to use or understand large numbers?

2. a. The national debt is about \$14,200,000,000,000. Write this number in scientific notation.

b. Suppose that there are 308,000,000 people in the United States. Write this number in scientific notation.

c. If the debt is divided equally among the people, how much (rounded to the nearest hundred dollars) is each person's share?

LEVEL 1 Essential Ideas

3. IN YOUR OWN WORDS What is an exponent?

4. IN YOUR OWN WORDS What is scientific notation?

5. IN YOUR OWN WORDS Describe a process for finding the prime factorization of a number.

6. IN YOUR OWN WORDS Describe the difference between the $\boxed{y^x}$ and $\boxed{\text{EE}}$ calculator keys.

7. Consider the number 10^6.

a. What is the common name for this number?

b. What is the base?

c. What is the exponent?

d. According to the definition of exponential notation, what does the number mean?

8. Consider the number 10^3.

a. What is the common name for this number?

b. What is the base?

c. What is the exponent?

d. According to the definition of exponential notation, what does the number mean?

9. Consider the number 10^{-1}.

a. What is the common name for this number?

b. What is the base?

c. What is the exponent?

d. According to the definition of exponential notation, what does the number mean?

10. Consider the number 10^{-2}.

a. What is the common name for this number?

b. What is the base?

c. What is the exponent?

d. According to the definition of exponential notation, what does the number mean?

LEVEL 1 Right or Wrong?

Explain what is wrong, if anything, with the statements in Problems 11-14. *Explain your reasoning.*

11. $5^2 = 10$

12. 2^3 means $2 + 2 + 2$

13. 4^3 means multiply 4 by itself 3 times

14. A number is in scientific notation when it is written as a number between 1 and 10 times a power of 10.

LEVEL 2 Drill and Practice

Write each of the numbers in Problems 15-20 *in scientific notation.*

15. a. 3,200 **b.** 25,000
c. 18,000,000 **d.** 640

16. a. 0.004
b. 0.02
c. 0.0035
d. 0.00000 045

17. a. 0.00000 421
b. 92,000,000
c. 1
d. $1\frac{1}{2}$

18. a. 0.00008 61
b. 249,000,000
c. 100
d. $11\frac{1}{2}$

19. a. 6.34 E9
b. 5.2019 E11
c. 4.093745 08
d. 8.291029292 12

20. a. 2.029283 − 03
b. 5.209 E − 05
c. 3.56 − 10
d. 3.8928 E − 14

Write each of the numbers in Problems 21-30 *without using exponents.*

21. a. 7.2×10^{10}
b. 4.5×10^3

22. a. 3.1×10^2
b. 6.8×10^8

23. a. 2.1×10^{-3}
b. 4.6×10^{-7}

24. a. 2.05×10^{-1}
b. 3.013×10^{-2}

25. a. 3.2×10^0
b. 8.03×10^{-4}

26. a. 5.06×10^3
b. 6.81×10^0

27. a. 7^2 **b.** 5^2

28. a. 4^5 **b.** 9^3

29. a. 2.18928271 10
b. 0.0000329 07

30. a. 0.00029214 E12
b. 4.29436732478 E19

Use the extended order of operations to simplify the expressions in Problems 31-36.

31. $2^3 + 5(4 + 3)$ **32.** $3(8 - 5) + 3^2$

33. $(2 + 5)^2 - 4^2$ **34.** $3^3 - (1 + 4)^2$

35. $(7 + 3 \cdot 2)^2$ **36.** $7 \quad + (3 \cdot 2)^2$

Find the prime factorization (written with exponents) for the numbers in Problems 37- 45.

37. a. 12 **b.** 20

38. a. 120 **b.** 24

39. a. 256 **b.** 18

40. a. 150 **b.** 105

41. a. 400 **b.** 1,000

42. a. 10,000 **b.** 720

43. a. 4,459
b. 229,333

44. a. 2,098,987
b. 803,257

45. a. 45,733
b. 29,791

Estimate answers for Problems 46-51 *by choosing the most reasonable answer. You should not do any pencil-and-paper or calculator arithmetic for these problems.*

46. The number of marbles that could be placed into a bathtub is about:
A. 10^3 B. 10^4 C. 10^9

47. The number of seconds since you have been born is about:
A. 10^9 B. 10^{70} C. 10^{700}

48. The cost of a pack (15 sticks) of gum (in dollars) is closest to:
A. 10^{-2} B. 10^{-1} C. 10^0

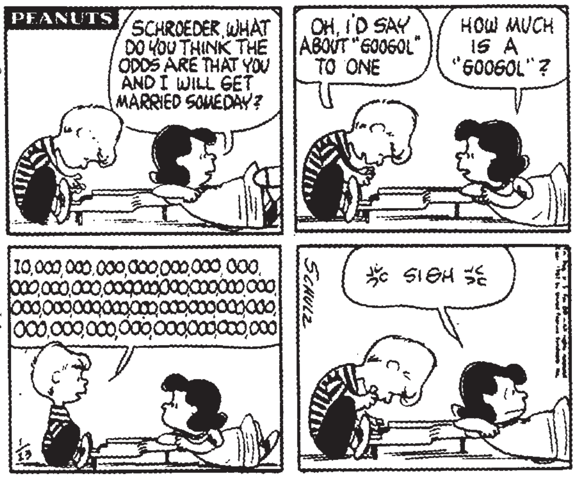
49. In a restaurant, the cost of a cup of coffee (in dollars) is closest to:
A. 10^{-2} B. 10^{-1} C. 10^0

50. The distance from the earth to the sun (in miles) is closest to:
A. 10^3 B. 10^7 C. 10^{18}

51. The number of grains of sand on the earth is closest to:
A. 10^{50} B. 10^{-50} C. 10^{100}

LEVEL 2 Applications

52. A **googol** is a very large number that is defined in the cartoon. Write a googol in scientific notation.



53. A thermochemical calorie is about 41,840,000 ergs. Write this number in scientific notation.

54. The estimated age of the earth is about 5×10^9 years. Write this number without using an exponent.

55. The mass of the sun is about 3.33×10^5 times the mass of the Earth. Write this without using an exponent.

56. a. The world's largest library, the Library of Congress, has approximately 59,000,000 items. Write this number in scientific notation.
b. If my local library has 54,000 volumes, how many times larger is the Library of Congress?

57. a. The sun develops about 5×10^{23} horsepower per second. Write this without using an exponent.
b. Convert this to horsepower per year.

58. a. A light-year is the distance that light travels in 1 year; this is about 5,869,713,600 miles. Write this distance in scientific notation.
b. If a spacecraft could travel at 82,300 mi/hr, how long would it take (in hours) for this craft to travel one light-year?
c. Estimate the time it would take (to the nearest year).

59. Estimate the total price of the following grocery items:

1 tube toothpaste, \$1.89
1 lb peaches, \$0.79
1 lb elbow macaroni, \$1.39
3 yogurts, \$1.61
lettuce, \$0.59
sliced turkey, \$2.50
chips, \$1.89
paper towels, \$0.99
half-gallon of orange juice, \$1.97
1 lb cheddar cheese, \$1.99
3 cucumbers, \$0.99
1 lb frozen corn, \$1.09
2 bars soap, \$1.79

60. Estimate the total cost of the following plumbing items:

120 $1\frac{1}{4}$-in. PVC pipe, \$76.80
$1\frac{1}{4}$-in. ball valve, \$17.89
$1\frac{1}{4}$-in PVC fittings, \$24.28
1-in. PVC fittings, \$14.16
$\frac{3}{4}$-in. PVC fittings, \$6.48
1-in. ball valve, \$12.84

1.6 Common Fractions

The POWER of Math

"Is Dr. Smith home?" asked Mary

"Just a minute—I'll check," answered Shannon. "Hey, Mom do you know if Dad is home?"

"Honey Bear, are you upstairs?" called out Shannon's mother.

In everyday life, we use different names or titles for ourselves, depending on the context or situation. For example, in some situations, I'm called Karl, in others Dr. Smith or Professor Smith, and in other places I'm Shannon's dad. We learn to deal with different names quite effectively in everyday situations, and we need to transfer this concept to numbers.

A common fraction may have different names or representations, each of which is better than others in a certain context. For example, $\frac{1}{2}$, 0.5, $\frac{50}{100}$, and $\frac{8}{16}$ all name the same common fraction. We call $\frac{1}{2}$ the **reduced form**, but any of these may be the preferred form depending on what we want to do. In this section, when we refer to a fraction, we are referring to a common fraction. We say that a fraction is **reduced** if no counting number other than 1 divides evenly into both the numerator and the denominator.

Reducing Fractions

The process of reducing a fraction relies on the **fundamental property of fractions** and on the process of factoring that we introduced in Section 1.5.

Fundamental Property of Fractions

If you multiply or divide both the numerator and the denominator by the same nonzero number, the resulting fraction will be the same.

STOP Reducing fractions is essential to working with fractions; study this procedure.

We will use this fundamental property first to reduce fractions and then to multiply fractions. Let's begin by stating the procedure for **reducing fractions**.

Reducing Fractions

To reduce a fraction:

Step 1 Find all common factors (other than 1) between the numerator and denominator.

Step 2 Divide out the common factors using the fundamental property of fractions.

EXAMPLE 1 Reduce a common fraction

Reduce $\frac{36}{48}$.

Solution

Step 1. Completely factor both numerator and denominator. We did this in the previous section.

Step 2. Write the numerator and denominator in factored form:

$$\frac{36}{48} = \frac{2 \times 2 \times 3 \times 3}{2 \times 2 \times 2 \times 2 \times 3}$$

Step 3. Use the fundamental property of fractions to eliminate the common factors. This process is sometimes called **canceling**.*

$$\frac{36}{48} = \frac{\cancel{2} \times \cancel{2} \times 3 \times \cancel{3}}{\cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{3}}$$

Step 4. Multiply the remaining factors. Treat the canceled factors as 1s.

$$1 \times 1 \times 3 \times 1 = 3$$

$$\frac{36}{48} = \frac{\cancel{2} \times \cancel{2} \times 3 \times \cancel{3}}{\cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{3}} = \frac{\mathbf{3}}{\mathbf{4}}$$ Notice how slashes are used as 1s.

$$1 \times 1 \times 2 \times 2 \times 1 = 4$$

This process is rather lengthy and can sometimes be shortened by noticing common factors that are larger than prime factors. For example, you might have noticed that 12 is a common factor in Example 1, so

$$\frac{36}{48} = \frac{3 \times \cancel{12}}{4 \times \cancel{12}} = \frac{3}{4}$$

Remember, this is 1.

EXAMPLE 2 Reducing fractions

Reduce the fractions.

a. $\frac{4}{8}$ **b.** $\frac{75}{100}$ **c.** $\frac{35}{55}$ **d.** $\frac{160}{180}$ **e.** $\frac{1,200}{9,000}$

Solution

a. $\frac{4}{8} = \frac{1 \times 4}{2 \times 4} = \frac{1}{2}$ **b.** $\frac{75}{100} = \frac{3 \times 25}{4 \times 25} = \frac{3}{4}$ **c.** $\frac{35}{55} = \frac{7 \times 5}{11 \times 5} = \frac{7}{11}$

If the fractions are complicated, you may reduce them in several steps.

d. $\frac{160}{180} = \frac{10 \times 16}{10 \times 18} = \frac{2 \times 8}{2 \times 9} = \frac{8}{9}$ **e.** $\frac{1,200}{9,000} = \frac{12 \times 100}{90 \times 100} = \frac{6 \times 2}{6 \times 15} = \frac{2}{15}$

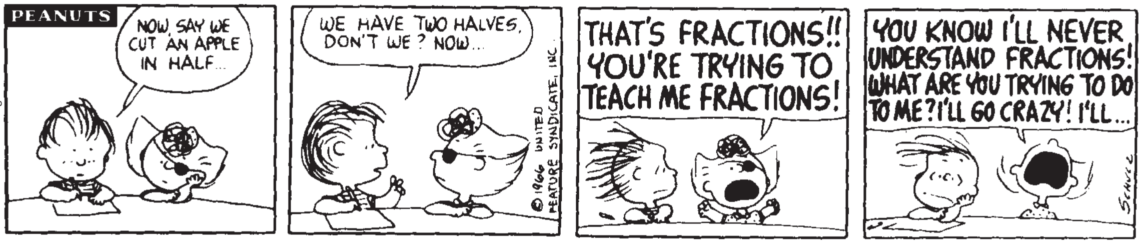
STOP In this book, all fractional answers should be reduced unless you are otherwise directed.

We say that a fraction is **completely reduced** when there are no common factors of both the numerator and the denominator.

We now include fractions as part of the simplification process. To **simplify a fractional expression** means to carry out all the operations, according to the order of operations, and to write the answer as a single number or a completely reduced fraction.

*Notice that *cancel* does not mean “cross out or delete” factors. It means “use the fundamental property to eliminate common factors.”

Multiplying Fractions



We now turn to multiplying fractions.

Multiplying Fractions

To **multiply fractions**, multiply numerators and multiply denominators.

EXAMPLE 3 Multiplying fractions

Simplify (that is, multiply the given numbers):

a. $\frac{1}{3} \times \frac{2}{5}$ **b.** $\frac{2}{3} \times \frac{4}{7}$ **c.** $5 \times \frac{2}{3}$ **d.** $3\frac{1}{2} \times 2\frac{3}{5}$

Solution

a. $\frac{1}{3} \times \frac{2}{5} = \boxed{\frac{1 \times 2}{3 \times 5}}$ ← *This step can often be done in your head.*

$= \frac{2}{15}$

b. $\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$

c. When multiplying a whole number and a fraction, as with $5 \times \frac{2}{3}$, write the whole number as a fraction and then multiply:

$$\frac{5}{1} \times \frac{2}{3} = \frac{10}{3} \text{ or } 3\frac{1}{3}$$

*Both $\frac{10}{3}$ (in part **c**) and $\frac{91}{10}$ (in part **d**) are reduced fractions. Recall that a fraction is reduced if there is no number (other than 1) that divides into both the numerator and denominator evenly.*

d. When multiplying mixed numbers, as with $3\frac{1}{2} \times 2\frac{3}{5}$, write the mixed numbers as improper fractions and then multiply:

$$\frac{7}{2} \times \frac{13}{5} = \frac{91}{10} \text{ or } 9\frac{1}{10}$$

Multiplication of fractions can be visualized by considering the meaning of fractions as shown in Figure 1.2.

Don't fall into the trap of saying, "Why do I need to do this? I can find $\frac{4}{5}$ of $\frac{2}{3}$ by $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$ and that's all there is to it!" Figure 1.2 shows why multiplication of fractions works the way it does.

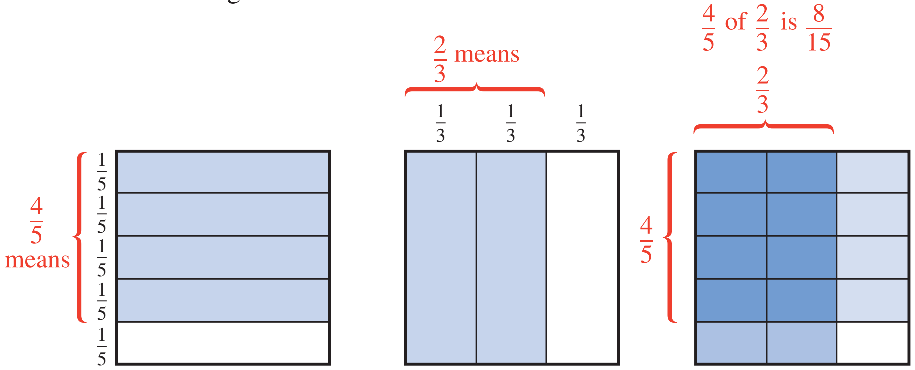
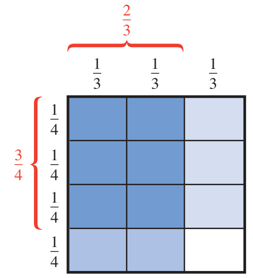


Figure 1.2 Geometrical justification of fractional multiplication

EXAMPLE 4 Illustrating the meaning of fractional multiplication

Show that $\frac{3}{4}$ of $\frac{2}{3}$ is $\frac{6}{12}$.

Solution



The result is $\frac{6}{12}$ (6 shaded parts out of 12 parts).

After the multiplication has been done, the product is often a fraction (such as the answer in Example 4) that can and should be reduced. Remember, a fraction is in reduced form if the only counting number that can divide evenly into both the numerator and the denominator is 1.

EXAMPLE 5 Multiplying fractions

Simplify (that is, multiply the following numbers):

a. $\frac{3}{4}\times\frac{2}{3}$ **b.** $\frac{2}{5}\times\frac{3}{4}$ **c.** $\frac{3}{5}\times\frac{1}{3}$ **d.** $3\frac{2}{3}\times 2\frac{2}{5}$ **e.** $\frac{4}{5}\times\frac{3}{8}\times\frac{2}{5}$

Solution

a. $\frac{3}{4}\times\frac{2}{3}=\frac{6}{12}$ but $\frac{6}{12}=\frac{\cancel{3}\times\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}\times\cancel{3}}=\frac{1}{2}$

Notice that the actual multiplication was a wasted step because the answer was simply factored again to reduce it! Therefore, the proper procedure is cancel common factors before you do the multiplication. Also, it is *not* necessary to write out prime factors. To reduce $\frac{6}{12}$, notice that 6 is a common factor.

These little numbers mean that → 6 divides into 6 one time, and 6 divides into 12 twice. → $\frac{\overset{1}{\cancel{6}}}{\underset{2}{\cancel{12}}}=\frac{1}{2}$ *6 is the common factor.*

b. $\frac{2}{5}\times\frac{3}{4}=\underbrace{\frac{\overset{1}{\cancel{2}}\times 3}{5\times\underset{2}{\cancel{4}}}}=\frac{3}{10}$

Notice that this is very similar to the original multiplication as stated to the left of the equal sign. You can save a step by canceling with the original product, as shown in the next part.

c. $\frac{\overset{1}{\cancel{3}}}{5}\times\frac{1}{\underset{1}{\cancel{3}}}=\frac{1}{5}$

d. $3\frac{2}{3}\times 2\frac{2}{5}=\frac{11}{\underset{1}{\cancel{3}}}\times\frac{\overset{4}{\cancel{12}}}{5}=\frac{44}{5}$ or $8\frac{4}{5}$

e. With a lengthier problem, such as $\frac{4}{5}\times\frac{3}{8}\times\frac{2}{5}$, you may do your canceling in several steps. We recopy the problem at each step for the sake of clarity, but in your work, the result would look like the last step only.

Step 1. $\frac{\overset{1}{\cancel{4}}}{5}\times\frac{3}{\underset{2}{\cancel{8}}}\times\frac{2}{5}$

Step 2. $\frac{\overset{1}{\cancel{4}}}{5}\times\frac{3}{\underset{1}{\cancel{\underset{\cancel{2}}{\cancel{8}}}}}\times\frac{\overset{1}{\cancel{2}}}{5}$

Step 3. $\frac{\overset{1}{\cancel{4}}}{5}\times\frac{3}{\underset{1}{\cancel{\underset{\cancel{2}}{\cancel{8}}}}}\times\frac{\overset{1}{\cancel{2}}}{5}=\frac{3}{25}$

When you want to find the fractional part of any number, you can do so by multiplication. That is, the word *of* is often translated into multiplication:

$$\tfrac{4}{5} \text{ of } \tfrac{2}{3} \text{ is } \quad \tfrac{4}{5} \times \tfrac{2}{3} = \tfrac{8}{15} \quad \text{and} \qquad \tfrac{3}{4} \text{ of } \tfrac{2}{3} \text{ is } \quad \tfrac{3}{4} \times \tfrac{2}{3} = \tfrac{6}{12} = \tfrac{1}{2}$$

The following example shows how you can use this idea with an applied problem.

EXAMPLE 6 Finding a sale price

If a store is having a "$\frac{1}{3}$ - *OFF*" sale, you must pay $\frac{2}{3}$ of the original price. What is the sale price of a suit that costs $355?

Solution We want to know "What is $\frac{2}{3}$ of $355?"

$$\frac{2}{3} \text{ of } 355 = \frac{2}{3} \times 355 = \frac{710}{3}$$

This mixed operation can easily be done by calculator:

$\boxed{2}\ \boxed{\times}\ \boxed{355}\ \boxed{\div}\ \boxed{3}\ \boxed{=}$

Since this is a problem involving a money answer, we want to write the answer in decimal form, rounded to the nearest cent. To do this, we divide 3 into 710 (and round) to obtain $236.67.

Dividing Fractions

If the product of two numbers is 1, then those numbers are called **reciprocals**. To find the reciprocal of a given number, write the number in fractional form and then **invert**, as shown in Example 7.

EXAMPLE 7 Finding reciprocals

Find the reciprocal of each number, and then prove that it is the correct reciprocal by multiplication.

Using a calculator, find the key labeled $\boxed{1/x}$ *or* $\boxed{x^{-1}}$ *to find the reciprocal of a number in the display. For Example 7b, the reciprocal of 3 can be found:* $\boxed{3}\ \boxed{1/x}$. *Calculator reciprocals are given as decimal fractions.*

Solution

a. $\frac{5}{11}$ — Invert for reciprocal: $\frac{11}{5}$ — *Check:* $\frac{5}{11} \times \frac{11}{5} = 1$

b. 3 — Write as a fraction: $\frac{3}{1}$ — Invert for reciprocal: $\frac{1}{3}$ — *Check:* $3 \times \frac{1}{3} = 1$

c. $2\frac{3}{4}$ — Write as a fraction: $\frac{11}{4}$ — Invert for reciprocal: $\frac{4}{11}$ — *Check:* $2\frac{3}{4} \times \frac{4}{11} = 1$

d. 0.2 — Write as a fraction: $\frac{2}{10} = \frac{1}{5}$ — Invert for reciprocal: $\frac{5}{1} = 5$ — *Check:* $0.2 \times 5 = 1$

e. Zero is the only whole number that does not have a reciprocal because any number multiplied by zero is zero (and not 1).

The process of division is very similar to the process of multiplication. To divide fractions, you must understand these three ideas:

- **How to multiply fractions**
- **Which term is called the *divisor***
- **How to find the reciprocal of a number**

In the expression $10 \div 5$, the divisor is 5; that is, the **divisor** is the quantity (5) by which the given number (10) is divided.

EXAMPLE 8 Finding the divisor

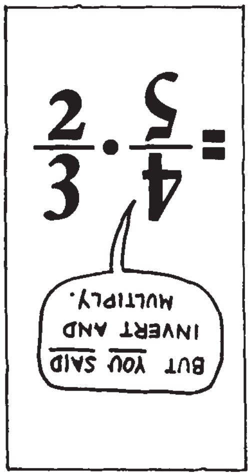
Name the divisor: **a.** $4 \div 2$ **b.** $2 \div 4$ **c.** $\frac{6}{5} \div \frac{2}{3}$ **d.** $\frac{0}{8}$ **e.** $\frac{8}{0}$

Solution **a.** 2 **b.** 4 **c.** $\frac{2}{3}$ **d.** 8 **e.** 0

Dividing Fractions

To **divide fractions**, multiply by the reciprocal of the divisor, This is sometimes phrased as "invert and multiply."

EXAMPLE 9 Dividing fractions



Cartoon courtesy of Patrick J. Boyle.

Simplify (that is, divide the following numbers):

a. $\frac{2}{3} \div \frac{4}{5}$ **b.** $\frac{3}{5} \div \frac{9}{20}$ **c.** $\frac{5}{8} \div 3$ **d.** $0 \div \frac{6}{7}$

Solution

a. $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$

$= \frac{\overset{1}{\cancel{2}}}{3} \times \frac{5}{\underset{2}{\cancel{4}}}$

$= \frac{5}{6}$

b. $\frac{3}{5} \div \frac{9}{20} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{4}{\cancel{20}}}{\underset{3}{\cancel{9}}}$

$= \frac{4}{3}$ or $1\frac{1}{3}$

c. $\frac{5}{8} \div 3 = \frac{5}{8} \times \frac{1}{3}$

$= \frac{5}{24}$

d. $0 \div \frac{6}{7} = 0 \times \frac{7}{6}$

$= 0$

Decimals to Fractions

We have discussed how to change a fraction to a decimal by division. The reverse procedure—changing a terminating decimal to a fraction—can be viewed as a multiplication problem involving fractions. This process is summarized in the following box.

Decimals to Fractions

Procedure for changing a terminating decimal into a fraction:

Step 1 Multiply the given number without its decimal point by the decimal name of the last digit.

Step 2 By the decimal name of the last digit, we mean:
One place is tenth, or $\frac{1}{10}$.
Two places is hundredth, or $\frac{1}{100}$.
Three places is thousandth, or $\frac{1}{1{,}000}$.
$\vdots$

EXAMPLE 10 Changing a terminating decimal to a fraction

Change the given decimal fractions to common fractions.

a. 0.4 **b.** 0.75 **c.** 0.0014 **d.** $0.12\frac{1}{2}$ **e.** $0.3\frac{1}{3}$

Solution

4 tenths; decimal position is tenth.

a. $0.4 = 4 \times \frac{1}{10} = \frac{\overset{2}{\cancel{4}}}{1} \times \frac{1}{\underset{5}{\cancel{10}}} = \frac{2}{5}$

75 hundredths; decimal position is hundredth.

b. $0.75 = \overset{3}{\cancel{75}} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{3}{4}$

14 ten-thousandths; decimal position is ten-thousandth.
↓

c. $0.0014 = \overset{7}{\cancel{14}} \times \dfrac{1}{\underset{5,000}{\cancel{10,000}}} = \dfrac{7}{5,000}$

$12\frac{1}{2}$ hundredths; decimal position is hundredth.
↓

d. $0.12\dfrac{1}{2} = 12\dfrac{1}{2} \times \dfrac{1}{100} = \dfrac{\overset{1}{\cancel{25}}}{2} \times \dfrac{1}{\underset{4}{\cancel{100}}} = \dfrac{1}{8}$

$3\ \frac{1}{3}$ tenths; decimal position is tenth.
↓

e. $0.3\dfrac{1}{3} = 3\dfrac{1}{3} \times \dfrac{1}{10} = \dfrac{\overset{1}{\cancel{10}}}{3} \times \dfrac{1}{\underset{1}{\cancel{10}}} = \dfrac{1}{3}$

Numbers in which decimal and fractional forms are mixed, as in parts **d** and **e** of Example 10, are called **complex decimals**.

Problem Set 1.6

The POWER of Math

1. **IN YOUR OWN WORDS** Give some examples of how you are perceived differently by different groups and associations in your life. What do you think this has to do with fractions?
2. **IN YOUR OWN WORDS** State the fundamental property of fractions, and explain why you think it is so "fundamental."

LEVEL 1 Essential Ideas

3. **IN YOUR OWN WORDS** Explain a procedure for reducing fractions.
4. **IN YOUR OWN WORDS** How do you know when a fraction is reduced?
5. **IN YOUR OWN WORDS** Explain a procedure for multiplying fractions.
6. **IN YOUR OWN WORDS** Explain a procedure for dividing fractions.
7. **IN YOUR OWN WORDS** What is a terminating decimal.
8. **IN YOUR OWN WORDS** Explain a procedure for changing a terminating decimal to a fraction.

LEVEL 1 Right or Wrong?

Explain what is wrong, if anything, with the statements in Problems 9-14. *Explain your reasoning.*

9. $\frac{9}{2}$ is a reduced fraction.

10. If I want to find $\frac{1}{2}$ of 16, the correct operation is to divide 16 by $\frac{1}{2}$.

11. $8 \times \dfrac{3}{2} = \dfrac{8 \times 3}{8 \times 2} = \dfrac{24}{16} = \dfrac{3}{2}$

12. To find the reciprocal of any number, write $\dfrac{1}{\text{the number}}$.

13. $\dfrac{2}{3} \div \dfrac{4}{5} = \dfrac{4}{5} \div \dfrac{2}{3}$

14. "Invert and multiply" means that a division problem can be carried out by changing the division to a multiplication, and then multiplying by the reciprocal of the divisor.

LEVEL 2 Drill and Practice

Completely reduce the fractions in Problems 15-20.

15. a. $\frac{2}{4}$ **b.** $\frac{3}{9}$ **c.** $\frac{4}{16}$ **d.** $\frac{2}{10}$

16. a. $\frac{14}{7}$ **b.** $\frac{38}{19}$ **c.** $\frac{92}{2}$ **d.** $\frac{160}{8}$

17. a. $\frac{72}{15}$ **b.** $\frac{42}{14}$ **c.** $\frac{16}{24}$ **d.** $\frac{128}{256}$

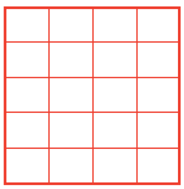
18. a. $\frac{18}{30}$ **b.** $\frac{70}{105}$ **c.** $\frac{50}{400}$ **d.** $\frac{35}{21}$

19. a. $\frac{140}{420}$ **b.** $\frac{75}{50}$ **c.** $\frac{240}{672}$ **d.** $\frac{5,670}{12,150}$

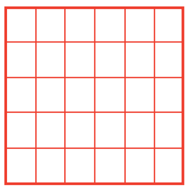
20. a. $\frac{12}{432}$ **b.** $\frac{150}{1,000}$ **c.** $\frac{2,500}{10,000}$ **d.** $\frac{105}{120}$

Illustrate the given parts of a whole in Problems 21-26.

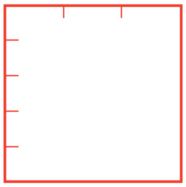
21. $\frac{2}{5}$ of $\frac{3}{4}$



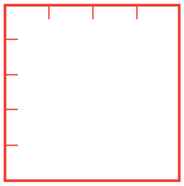
22. $\frac{3}{5}$ of $\frac{5}{6}$



23. $\frac{4}{5}$ of $\frac{1}{3}$



24. $\frac{3}{5}$ of $\frac{3}{4}$



25. $\frac{1}{6}$ of $\frac{2}{3}$

26. $\frac{3}{5}$ of $\frac{1}{2}$

In Problems 27-30, first name the divisor, and then find the answer by direct division. Finally, find the answer by multiplying by the reciprocal. Both answers should be the same.

27. a. $15 \div 5$ **b.** $6 \div 3$

28. a. $0 \div 3$ **b.** $14 \div 2$

29. a. $20 \div 4$ **b.** $240 \div 16$

30. a. $5 \div 0.2$ **b.** $2.4 \div 0.6$

Simplify the numerical expressions given in Problems 31-41. Give your answers in reduced form.

31. a. $\frac{1}{4} \times \frac{1}{6}$ **b.** $\frac{2}{3} \times \frac{4}{5}$ **c.** $\frac{3}{4} \times \frac{1}{6}$
d. $\frac{7}{20} \times \frac{100}{14}$ **e.** $\frac{5}{9} \times \frac{18}{25}$ **f.** $\frac{2}{3} \times \frac{3}{8}$

32. a. $\frac{4}{5} \times \frac{13}{16}$ **b.** $\frac{18}{25} \times \frac{5}{36}$ **c.** $\frac{4}{5} \times \frac{3}{8}$
d. $\frac{4}{7} \times \frac{14}{9}$ **e.** $\frac{5}{12} \times \frac{4}{15}$ **f.** $\frac{9}{16} \times \frac{4}{27}$

33. a. $\frac{1}{2} \div \frac{1}{3}$ **b.** $\frac{1}{3} \div \frac{1}{2}$ **c.** $\frac{2}{3} \div \frac{1}{2}$
d. $\frac{3}{4} \div \frac{2}{3}$ **e.** $\frac{2}{5} \times \frac{15}{8}$ **f.** $\frac{5}{3} \times \frac{9}{15}$

34. a. $\frac{3}{5} \times \frac{20}{27}$ **b.** $\frac{3}{8} \div \frac{15}{16}$ **c.** $\frac{2}{3} \div \frac{5}{6}$
d. $\frac{4}{5} \div \frac{3}{10}$ **e.** $\frac{4}{7} \div \frac{4}{5}$ **f.** $\frac{5}{6} \div \frac{1}{3}$

35. a. $\frac{4}{5} \div \frac{4}{5}$ **b.** $\frac{7}{9} \div \frac{7}{9}$ **c.** $\frac{8}{3} \div \frac{8}{3}$
d. $\frac{4}{5} \times 5$ **e.** $\frac{3}{5} \div \frac{3}{7}$ **f.** $\frac{6}{7} \div \frac{2}{3}$

36. a. $\frac{4}{9} \div \frac{3}{4}$ **b.** $\frac{2}{3} \times 3$ **c.** $\frac{5}{6} \times 18$
d. $\frac{3}{8} \times 24$ **e.** $\frac{5}{8} \times 8$ **f.** $\frac{6}{7} \times 7$

37. a. $\frac{2}{5} \div 3$ **b.** $\frac{3}{8} \div 3$ **c.** $\frac{3}{5} \div 5$
d. $3 \div \frac{1}{6}$ **e.** $2\frac{1}{2} \div 3$ **f.** $6\frac{1}{2} \div 3$

38. a. $3\frac{4}{5} \div 0$ **b.** $7 \times \frac{9}{14}$
c. $5 \div 1\frac{1}{2}$ **d.** $4 \div 2\frac{2}{3}$
e. $6 \div 1\frac{5}{6}$ **f.** $52 \times \frac{5}{13}$

39. a. $2\frac{2}{5} \times 1\frac{4}{5}$ **b.** $5\frac{1}{2} \times 3\frac{2}{3}$
c. $4\frac{1}{6} \times 3\frac{3}{8}$ **d.** $1\frac{1}{6} \times 2\frac{1}{3}$
e. $6\frac{1}{2} \times \frac{5}{6}$ **f.** $3\frac{4}{5} \times \frac{1}{2}$

40. a. $2\frac{2}{5} \div 1\frac{2}{3}$ **b.** $5\frac{1}{2} \div 1\frac{4}{5}$
c. $2\frac{2}{3} \div 1\frac{1}{3}$ **d.** $5 \times \frac{3}{5}$
e. $2\frac{1}{2} \times \frac{3}{4}$ **f.** $4\frac{1}{2} \div 9\frac{1}{2}$

41. a. $\frac{1}{2} \times \frac{8}{9} \times \frac{3}{16}$ **b.** $\frac{2}{3} \times \frac{5}{8} \times \frac{16}{100}$
c. $\frac{2}{3} \times \frac{4}{5} \times \frac{15}{16}$ **d.** $2\frac{1}{2} \times 3\frac{1}{6} \times 1\frac{1}{5}$
e. $\left(\frac{1}{2} \div \frac{1}{2}\right) \div \frac{1}{4}$ **f.** $\frac{1}{2} \div \left(\frac{1}{3} \div \frac{1}{4}\right)$

Write the decimals in Problems 42-49 in fractional form.

42. a. 0.7 **b.** 0.9 **c.** 0.8

43. a. 0.25 **b.** 0.87 **c.** 0.375

44. a. 0.18 **b.** 0.48 **c.** 0.54

45. a. 0.78 **b.** 0.85 **c.** 0.246

46. a. 0.505 **b.** 0.015 **c.** 0.005

47. a. $0.66\frac{2}{3}$ **b.** $0.87\frac{1}{2}$ **c.** $0.16\frac{2}{3}$

48. a. $0.37\frac{1}{2}$ **b.** $0.8\frac{8}{9}$ **c.** $0.000\frac{1}{3}$

49. a. $0.1\frac{1}{9}$ **b.** $0.5\frac{5}{9}$ **c.** $0.08\frac{1}{3}$

LEVEL 2 Applications

Some experts tell us that the amount of savings we need is $\frac{1}{10}$ times our age times our current salary. Calculate the amount of savings necessary in Problems 50-53.

50. age 20, salary \$24,000

51. age 30, salary \$46,000

52. age 25, salary \$25,000

53. age 50, salary \$125,000

54. If two-thirds of a person's body weight is water, what is the weight of water in a person who weighs 180 pounds?

55. If Karl owns $\frac{3}{16}$ of a mutual water system and a new well is installed at a cost of \$12,512, how much does Karl owe for his share?

56. What would 100 shares of Xerox stock cost when it is selling for \$56.63 per share?

57. If you received \$227.00 for selling 20 shares of Brunswick stock, what was the price per share?

58. A recipe calls for $\frac{5}{8}$ cup of sugar, one egg, and $\frac{7}{8}$ cup of flour. How much of each ingredient is needed to double this mixture?

59. If $4\frac{2}{5}$ acres sell for \$44,000, what is the price per acre?

60. The following advertisement recently appeared in a Kansas newspaper: "Divide your age by one-half and that is the percent discount you will receive for this sale!" Interpret and critique this statement.

1.7 Adding and Subtracting Fractions

The POWER of Math

"Darn! Every time I cut one of these buggers, I'm off by a sixteenth of an inch," yelled Jim, the carpenter's apprentice.

Arnie smiled as he powered down his saw. "You're forgetting the width of the cut, youngin'. When you're making more than one cut, you need to add the thickness of the blade into your measurement."

"Ah, yeah," admitted Jim, scratching his head. "Forgot about that—no wonder my cabinet looks deformed. It's amazing how much a sixteenth of an inch can throw something off."

In this section, you will learn how to work with fractions—namely, adding and subtracting them.

Common Denominators

If the fractions you are adding or subtracting are similar (all halves, thirds, fourths, fifths, sixths, and so on), then the procedure is straightforward: Add or subtract the numerators. In this case, we say that the fractions have **common denominators**. If the fractions are not similar, then you *cannot* add or subtract them directly; you must change the form of the fractions so that they are similar. This process is called *finding common denominators.*

Adding/Subtracting Fractions

To **add** or **subtract fractions** with common denominators, add or subtract the numerators. The denominator of the sum or difference is the same as the common denominator.

EXAMPLE 1 Adding and subtracting fractions with common denominators

© Greg Ceo/Stone/Getty Images

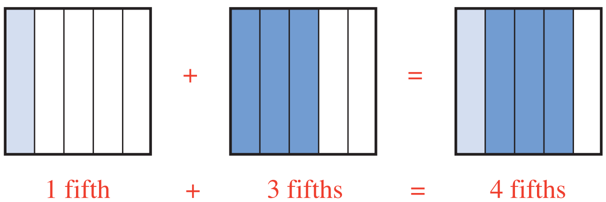
Simplify the given numerical expressions.

a. $\frac{1}{5} + \frac{3}{5}$ **b.** $\frac{5}{9} - \frac{4}{9}$

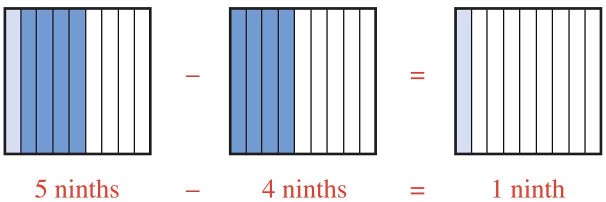
c. $\frac{3}{2} + \frac{7}{2}$ **d.** $3\frac{2}{3} + 1\frac{2}{3} + \frac{1}{3} + 4\frac{2}{3}$

Solution

a. $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$



b. $\frac{5}{9} - \frac{4}{9} = \frac{1}{9}$



c. $\frac{3}{2} + \frac{7}{2} = \frac{10}{2} = 5$ This example could also be added in the form of mixed numbers:

Add fraction part first.

$$\begin{array}{r} 1\frac{1}{2} \\ +3\frac{1}{2} \\ \hline 4\frac{2}{2} \end{array}$$

Next, add whole number part.

Since $\frac{2}{2} = 1$, it follows that $4\frac{2}{2} = 4 + \frac{2}{2} = 4 + 1 = 5$.

d.

$$\begin{array}{r} 3\frac{2}{3} \\ 1\frac{2}{3} \\ \frac{1}{3} \\ +\ 4\frac{2}{3} \\ \hline 8\frac{7}{3} = 10\frac{1}{3} \end{array}$$

Notice that the carry may be more than 1. In this example, $\frac{7}{3} = 2\frac{1}{3}$.

Sometimes when doing subtraction, you must borrow from the units column to have enough fractional parts to carry out the subtraction. Remember,

CAUTION

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7} = \frac{8}{8} = \frac{9}{9} = \frac{10}{10} = \cdots$$

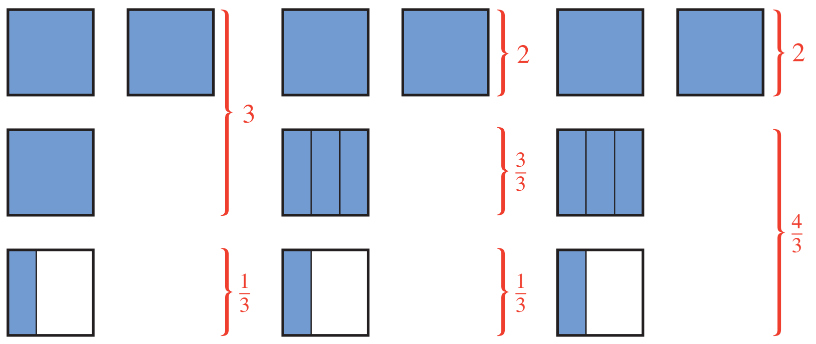
EXAMPLE 2 Subtraction with common denominators and borrowing

Borrow 1 from the units column and combine with the fractional part.

a. $3\frac{1}{3}$ **b.** $5\frac{3}{5}$ **c.** $3\frac{1}{2}$ **d.** $1\frac{1}{3}$ **e.** $14\frac{4}{5}$

Solution

a. $3\frac{1}{3}$: You would do the following steps in your head and write only the answer:



CAUTION Study this to understand rewriting numbers to carry out subtraction with mixed numbers and borrowing.

These all represent the same number: $3 + \frac{1}{3}$ $2 + \frac{3}{3} + \frac{1}{3}$ $2 + \frac{4}{3}$

Thus, $3\frac{1}{3} = 2\frac{4}{3}$.

b. $5\frac{3}{5} = 4\frac{8}{5}$ **c.** $3\frac{1}{2} = 2\frac{3}{2}$ **d.** $1\frac{1}{3} = \frac{4}{3}$ **e.** $14\frac{4}{5} = 13\frac{9}{5}$

Using the idea shown in Example 2, we can carry out some subtractions with mixed numbers.

EXAMPLE 3 Subtraction of mixed numbers with common denominators

Simplify the given numerical expressions.

a. $3\frac{1}{3} - 1\frac{2}{3}$ **b.** $4\frac{3}{5} - 2\frac{4}{5}$ **c.** $1\frac{1}{3} - \frac{2}{3}$

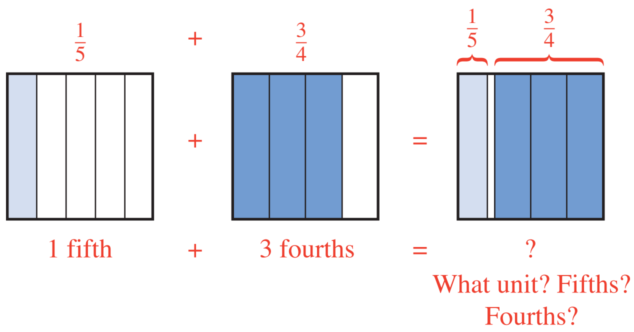
Solution

a.
$$\begin{array}{rcr} 3\frac{1}{3} & = & 2\frac{4}{3} \\ -\;1\frac{2}{3} & = & -\;1\frac{2}{3} \\ \hline & & 1\frac{2}{3} \end{array}$$

b.
$$\begin{array}{rcr} 4\frac{3}{5} & = & 3\frac{8}{5} \\ -\;2\frac{4}{5} & = & -\;2\frac{4}{5} \\ \hline & & 1\frac{4}{5} \end{array}$$

c.
$$\begin{array}{rcr} 1\frac{1}{3} & = & \frac{4}{3} \\ -\;\frac{2}{3} & = & -\;\frac{2}{3} \\ \hline & & \frac{2}{3} \end{array}$$

If the fractions to be added or subtracted do not have common denominators, the sum or difference is not as easy to find. For example, consider the sum $\frac{1}{5} + \frac{3}{4}$.



Lowest Common Denominator

We need to use the fundamental property of fractions to change the form of one or more of the fractions so that they do have the same denominators. Use the following guidelines to find the best common denominator.

Pay attention here:

First: The common denominator should be a number into which all the given denominators divide evenly. This means that the product of the given denominators will always be a common denominator. Many students learn this and *always* find the common denominator by multiplication. Even though this works for all numbers, it is inefficient except for small numbers. Therefore, we have a second condition to find the best common denominator.

Second: The common denominator should be as small as possible. This number is called the **lowest common denominator**, denoted by **LCD**.

Lowest Common Denominator

The procedure for finding the lowest common denominator (LCD) is:

Step 1 Factor all given denominators into prime factors; write each factorization using exponents.

Step 2 List each different prime factor you found in the prime factorization of the denominator.

Step 3 On each prime in the list from step 2, place the largest exponent that appears on that prime factor anywhere in the factorization of the denominators.

Step 4 The LCD is the product of the prime factors with the exponents in step 3.

EXAMPLE 4 Finding the lowest common denominator

Find the LCD for the given denominators.

a. 6 and 8 **b.** 8 and 12 **c.** 24 and 30 **d.** 8, 24, and 60 **e.** 300 and 144

Solution

a. 6; 8

$6 = 2 \times 3$

$8 = 2^3$ Largest exponent on prime factor 3 is 1 (remember, $3 = 3^1$).

LCD: $2^3 \times 3 = 8 \times 3 = 24$

Largest exponent on prime factor 2 is 3.

b. 8; 12

$8 = 2^3$

$12 = 2^2 \times 3$

LCD: $2^3 \times 3 = 8 \times 3 = 24$

c. 24; 30

$24 = 2^3 \times 3$

$30 = 2 \times 3 \times 5$

LCD: $2^3 \times 3 \times 5 = 8 \times 3 \times 5 = 120$

d. 8; 24; 60 The same procedure works no matter how many denominators are given.

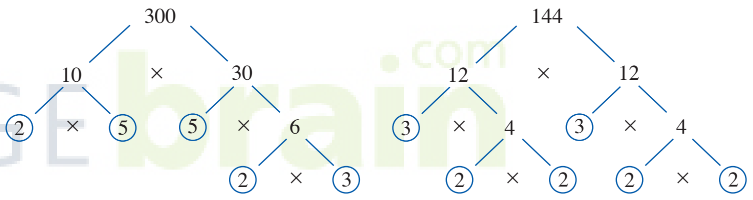
$8 = 2^3$

$24 = 2^3 \times 3$

$60 = 2^2 \times 3 \times 5$

LCD: $2^3 \times 3 \times 5 = 8 \times 3 \times 5 = 120$

e. 300; 144 If the numbers are larger, you may need factor trees:



300 $= 2^2 \times 3 \times 5^2$

144 $= 2^4 \times 3^2$

LCD: $2^4 \times 3^2 \times 5^2 = 16 \times 9 \times 25 = 3{,}600$

Procedure for Adding and Subtracting Fractions

We now turn to adding and subtracting fractions by finding the lowest common denominator.

Adding/Subtracting

To add or subtract fractions that do not have common denominators, carry out the following procedure:

Step 1 Find the LCD

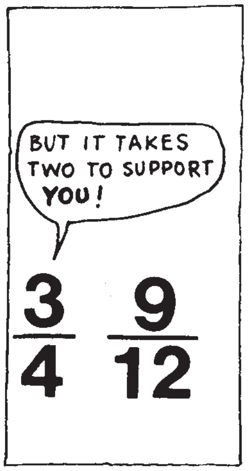
Step 2 Change the forms of the fractions to obtain forms with common denominators.

Step 3 Add or subtract the numerators of the fractions with common denominators.

EXAMPLE 5 Addition and subtraction of fractions

Simplify:

a. $\frac{1}{6}+\frac{3}{4}$ **b.** $\frac{4}{9}-\frac{1}{4}$ **c.** $2\frac{1}{6}+5\frac{3}{8}$ **d.** $12\frac{7}{24}-5\frac{4}{30}$ **e.** $\frac{3}{8}+\frac{11}{12}+\frac{3}{20}$ **f.** $16\frac{1}{3}-4\frac{1}{2}$



Cartoon courtesy of Patrick J. Boyle.

Solution

a. Write in column form:

$$\begin{array}{r} \frac{1}{6} \\ +\ \frac{3}{4} \\ \hline \end{array} \qquad \begin{array}{lll} \text{Find the LCD:} & 6 & = 2 \times 3 \\ & 4 & = 2^2 \\ \text{LCD:} & & 2^2 \times 3 = 12 \end{array}$$

Next, change form:

$$\begin{array}{rcl} \frac{1}{6} & = & \frac{2}{12} \\ +\ \frac{3}{4} & = & \frac{9}{12} \\ \hline \text{Add:} & & \frac{11}{12} \end{array} \qquad \begin{array}{l} \text{Mult by 1: } \frac{1}{6} \times \frac{2}{2} = \frac{2}{12} \\ \text{Again: } \frac{3}{4} \times \frac{3}{3} = \frac{9}{12} \end{array}$$

In this example, you are asked to write

$$\frac{3}{4} \quad \text{as} \quad \frac{9}{12}$$

and we are reminded of the cartoon in the margin.

b. Combining the steps outlined in part **a**, you'll obtain

$$\begin{array}{rcr} \frac{4}{9} & = & \frac{16}{36} \\ -\ \frac{1}{4} & = & -\ \frac{9}{36} \\ \hline & & \frac{7}{36} \end{array} \qquad \begin{array}{lll} \text{Find the LCD:} & 9 & = 3^2 \\ & 4 & = 2^2 \\ \text{LCD:} & & 2^2 \times 3^2 = 36 \end{array}$$

c.
$$\begin{array}{rcr} 2\frac{1}{6} & = & 2\frac{4}{24} \\ +\ 5\frac{3}{8} & = & 5\frac{9}{24} \\ \hline & & 7\frac{13}{24} \end{array} \qquad \begin{array}{lll} \text{Find the LCD:} & 6 & = 2 \times 3 \\ & 8 & = 2^3 \\ \text{LCD:} & & 2^3 \times 3 = 24 \end{array}$$

d.
$$\begin{array}{rcr} 12\frac{7}{24} & = & 12\frac{35}{120} \\ -\ 5\frac{4}{30} & = & -\ 5\frac{16}{120} \\ \hline & & 7\frac{19}{120} \end{array} \qquad \begin{array}{lll} \text{Find the LCD:} & 24 & = 2^3 \times 3 \\ & 30 & = 2 \times 3 \times 5 \\ \text{LCD:} & & 2^3 \times 3 \times 5 = 120 \end{array}$$

e.
$$\begin{array}{rcl} \frac{3}{8} & = & \frac{45}{120} \\ \frac{11}{12} & = & \frac{110}{120} \\ +\ \frac{3}{20} & = & \frac{18}{120} \\ \hline & & \frac{173}{120} = 1\frac{53}{120} \end{array} \qquad \begin{array}{lll} \text{Find the LCD:} & 8 & = 2^3 \\ & 12 & = 2^2 \times 3 \\ & 20 & = 2^2 \times 5 \\ \text{LCD:} & & 2^3 \times 3 \times 5 = 120 \end{array}$$

Borrow so that you can complete the subtraction.

f. $$\begin{array}{rcrcr} 16\frac{1}{3} & = & 16\frac{2}{6} & = & 15\frac{8}{6} \\ -\ 4\frac{1}{2} & = & -\ 4\frac{3}{6} & = & -\ 4\frac{3}{6} \\ \hline & & & & 11\frac{5}{6} \end{array}$$

The order of operations is the same for fractions as it is for whole numbers.

EXAMPLE 6 Mixed operations with fractions

Simplify: **a.** $\frac{3}{4} \times \frac{1}{3} + \frac{3}{4} \times \frac{2}{3}$ **b.** $\frac{3}{4}\left(\frac{1}{3} + \frac{2}{3}\right)$

Solution Remember the correct order of operations.

a. $$\frac{3}{4} \times \frac{1}{3} + \frac{3}{4} \times \frac{2}{3} = \frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

b. $$\frac{3}{4}\left(\frac{1}{3} + \frac{2}{3}\right) = \frac{3}{4} \times 1 = \frac{3}{4}$$

Recall that juxtaposition implies multiplication.

The distributive property tells us that the answers to parts **a** and **b** in Example 6 must be equal.

EXAMPLE 7 Mixed operations using juxtaposition for multiplication

Simplify: **a.** $\frac{5}{6}\left(\frac{3}{4} + \frac{1}{2}\right) + \frac{2}{3} \times \frac{9}{10}$ **b.** $\frac{1}{2} \div \frac{1}{3} \div \left(\frac{3}{4} + \frac{3}{5}\right)$

Solution

a. $$\frac{5}{6}\left(\frac{3}{4} + \frac{1}{2}\right) + \frac{2}{3} \times \frac{9}{10} = \frac{5}{6}\left(\frac{3}{4} + \frac{2}{4}\right) + \frac{2}{3} \times \frac{9}{10}$$ Common denominator, parentheses first

$$= \frac{5}{6}\left(\frac{5}{4}\right) + \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{5}{\cancel{10}}}$$

$$= \frac{25}{24} + \frac{3}{5}$$

$$= \frac{125}{120} + \frac{72}{120}$$

$$= \frac{197}{120} \text{ or } 1\frac{77}{120}$$

b. $$\frac{1}{2} \div \frac{1}{3} \div \left(\frac{3}{4} + \frac{3}{5}\right) = \frac{1}{2} \div \frac{1}{3} \div \left(\frac{15}{20} + \frac{12}{20}\right)$$

$$= \frac{1}{2} \div \frac{1}{3} \div \frac{27}{20}$$

$$= \frac{1}{2} \times \frac{3}{1} \div \frac{27}{20}$$ Multiplication and division from left to right

$$= \frac{3}{2} \div \frac{27}{20}$$

$$= \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{10}{\cancel{20}}}{\underset{9}{\cancel{27}}}$$

$$= \frac{10}{9} \text{ or } 1\frac{1}{9}$$

Problem Set 1.7

The POWER of Math

1. **IN YOUR OWN WORDS** Describe a procedure for adding fractions that do not have common denominators.
2. **IN YOUR OWN WORDS** Describe a procedure for subtracting fractions that do not have common denominators.
3. Suppose you are installing molding around a table and you need pieces that are $5\frac{1}{4}$ in., $7\frac{1}{2}$ in., and $5\frac{3}{16}$ in. long. If the saw chews up $\frac{1}{16}$ in. of material each time a cut is made, what is the smallest single length of molding that can be used to do this job?
4. If the outside diameter of a piece of tubing is $\frac{15}{16}$ in. and the wall is $\frac{3}{16}$ in. thick, what is the inside diameter?

LEVEL 1 Essential Ideas

5. **IN YOUR OWN WORDS** What is the extended order-of-operations agreement?
6. **IN YOUR OWN WORDS** State a procedure for finding the lowest common denominator for a set of fractions.

LEVEL 1 Right or Wrong?

Explain what is wrong, if anything, with the statements in Problems 7-12. *Explain your reasoning.*

7. $\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8+8} = \frac{7}{16}$
8. $3\frac{5}{8} - 2\frac{7}{8} = 1\frac{2}{8} = 1\frac{1}{4}$
9. $\frac{3}{8} \times \frac{5}{8} = \frac{3 \times 5}{8 \times 8} = \frac{15}{64}$
10. $\frac{3}{8} \div \frac{5}{8} = \frac{8}{3} \times \frac{5}{8} = \frac{5}{3}$
11. $\frac{3}{8} + \frac{2}{5} = \frac{3 \times 5 + 2 \times 8}{8 \times 5} = \frac{15+16}{40} = \frac{31}{40}$
12. $\frac{3}{5} + \frac{2}{5} \times \frac{1}{2} = 1 \times \frac{1}{2} = \frac{1}{2}$

LEVEL 2 Drill and Practice

Estimate (do not calculate) the answers in Problems 13-20.

13. $\frac{19}{40}$ is about the same as
    A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{5}{4}$ D. $\frac{1}{5}$
14. $\frac{3}{10}$ is about the same as
    A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{5}{4}$ D. $\frac{1}{5}$
15. $\frac{19}{40} - \frac{3}{10}$ is about the same as
    A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{5}{4}$ D. $\frac{1}{5}$
16. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$ is about equal to
    A. $\frac{1}{2}$ B. 1 C. $\frac{3}{2}$ D. 1,000
17. If I multiply a given nonzero number by $\frac{2}{3}$, the answer _____ than the given number.
    A. is larger
    B. is smaller
    C. could be either larger or smaller
18. If I multiply a given nonzero number by $\frac{3}{2}$, the answer _____ than the given number.
    A. is larger
    B. is smaller
    C. could be either larger or smaller
19. If I divide a given nonzero number by $\frac{2}{3}$, the answer _____ than the given number.
    A. is larger
    B. is smaller
    C. could be either larger or smaller
20. If I divide a given nonzero number by $\frac{3}{2}$, the answer _____ than the given number.
    A. is larger
    B. is smaller
    C. could be either larger or smaller

Simplify the numerical expressions given in Problems 21-23. *Give your answers in reduced form.*

21. **a.** $\frac{2}{5} + \frac{1}{5}$ **b.** $\frac{3}{7} + \frac{5}{7}$ **c.** $\frac{5}{11} + \frac{3}{11}$
    d. $\frac{3}{2} + \frac{5}{2}$ **e.** $\frac{7}{3} - \frac{4}{3}$ **f.** $\frac{5}{9} + \frac{1}{9}$
22. **a.** $\frac{9}{13} - \frac{5}{13}$ **b.** $\frac{6}{23} - \frac{5}{23}$ **c.** $\frac{9}{7} - \frac{2}{7}$
    d. $\frac{13}{15} + \frac{2}{15}$ **e.** $\frac{7}{12} - \frac{1}{12}$ **f.** $\frac{5}{8} - \frac{3}{8}$
23. **a.** $\begin{array}{r} 2\frac{2}{3} \\ +\ 1\frac{1}{3} \\ \hline \end{array}$ **b.** $\begin{array}{r} 6\frac{4}{5} \\ +\ 2\frac{1}{5} \\ \hline \end{array}$ **c.** $\begin{array}{r} 8\frac{3}{8} \\ +\ 4\frac{5}{8} \\ \hline \end{array}$

d. $\begin{array}{r} 5\frac{1}{3} \\ -\ 3\frac{2}{3} \\ \hline \end{array}$ **e.** $\begin{array}{r} 14\frac{1}{8} \\ -\ 8\frac{5}{8} \\ \hline \end{array}$ **f.** $\begin{array}{r} 6\frac{1}{4} \\ -\ 5\frac{3}{4} \\ \hline \end{array}$

Find the LCD for the denominators given in Problems 24-27.

24. a. 4; 8 **b.** 2; 6
c. 2; 5 **d.** 5; 12

25. a. 4; 12 **b.** 12; 90
c. 12; 336 **d.** 90; 210

26. a. 6; 8; 10 **b.** 9; 12; 14
c. 60; 18 **d.** 450; 15

27. a. 735; 1,125 **b.** 315; 735
c. 420; 450 **d.** 600; 90; 30

Simplify the numerical expressions given in Problems 28-39. *Remember that fractions are not considered simplified unless they are reduced.*

28. a. $\frac{1}{2} + \frac{2}{3}$ **b.** $\frac{1}{2} + \frac{3}{8}$
c. $\frac{1}{2} - \frac{1}{6}$ **d.** $\frac{1}{2} + \frac{2}{5}$
e. $\frac{1}{2} - \frac{2}{5}$ **f.** $\frac{5}{6} + \frac{2}{3}$

29. a. $\frac{5}{6} - \frac{1}{3}$ **b.** $\frac{5}{6} + \frac{5}{8}$
c. $\frac{5}{8} - \frac{1}{3}$ **d.** $\frac{3}{4} - \frac{5}{12}$
e. $\frac{3}{5} + \frac{1}{6}$ **f.** $\frac{3}{4} + \frac{1}{12}$

30. a. $\frac{2}{45} + \frac{1}{6}$ **b.** $\frac{41}{45} - \frac{5}{6}$
c. $\frac{4}{9} - \frac{5}{12}$ **d.** $\frac{3}{5} + \frac{1}{12}$
e. $\frac{5}{24} - \frac{2}{15}$ **f.** $\frac{5}{27} + \frac{1}{90}$

31. a. $\begin{array}{r} 2\frac{1}{2} \\ +\ 4\frac{3}{4} \\ \hline \end{array}$ **b.** $\begin{array}{r} 1\frac{2}{3} \\ +\ 5\frac{1}{2} \\ \hline \end{array}$ **c.** $\begin{array}{r} 3\frac{3}{8} \\ +\ 5\frac{1}{2} \\ \hline \end{array}$

32. a. $\begin{array}{r} 5\frac{1}{8} \\ -\ 3\frac{3}{4} \\ \hline \end{array}$ **b.** $\begin{array}{r} 17\frac{1}{2} \\ -\ 6\frac{2}{3} \\ \hline \end{array}$ **c.** $\begin{array}{r} 12\frac{1}{3} \\ -\ 4\frac{1}{2} \\ \hline \end{array}$

33. a. $\frac{1}{8} + 2\frac{2}{3} + \frac{1}{6}$
b. $\frac{4}{5} + \frac{3}{7} + \frac{3}{10}$

34. a. $6\frac{1}{8} + 3\frac{2}{5} + 1\frac{1}{4}$
b. $5\frac{1}{2} + 6\frac{3}{4} + 4\frac{1}{8}$

35. a. $7\frac{2}{3} + 5\frac{1}{2} + 12\frac{1}{6}$
b. $12\frac{3}{8} + 2\frac{1}{2} + 5\frac{1}{8}$

36. a. $\frac{1}{2} + \frac{2}{3} \times \frac{1}{5}$
b. $\frac{1}{2} \div \left(\frac{1}{3} \div \frac{1}{4}\right)$

37. a. $\frac{1}{2} \times \frac{2}{3} + \frac{1}{5}$
b. $\frac{1}{5} \div \frac{1}{3} \div \frac{1}{4}$

38. a. $\frac{3}{4}\left(\frac{9}{13} + \frac{4}{13}\right)$
b. $\frac{4}{5}\left(\frac{5}{16} + \frac{11}{16}\right)$

39. a. $\dfrac{3 \times 3 + 5 \times 2}{5 \times 3}$
b. $\dfrac{3 \times 5 + 7 \times 4}{7 \times 5}$

Perform the indicated operations in Problems 40-45. *You may use your calculator and leave answers in decimalform.*

40. $\frac{3}{4} \times \frac{119}{200} + \frac{3}{4} \times \frac{81}{200}$

41. $\frac{4}{5} \times \frac{17}{95} + \frac{4}{5} \times \frac{78}{95}$

42. $\frac{7}{8} \times \frac{107}{147} + \frac{7}{8} \times \frac{40}{147}$

43. $\left(\frac{4}{5} + \frac{2}{3}\right) \div \frac{1}{5} + 2$

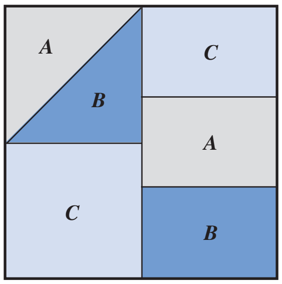
44. $\frac{19}{300} + \frac{55}{144} + \frac{25}{108}$

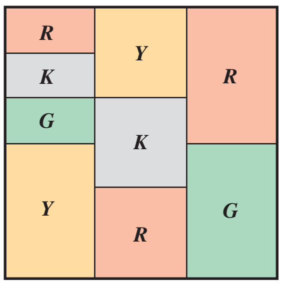
45. $\frac{15}{253} + \left(\frac{53}{104} - \frac{25}{208}\right)$

LEVEL 2 Applications

Estimate the portion of each square occupied by the indicated letter in Problems 46-57.

46. *A*
47. *B*
48. *C*
49. *A* or *B*
50. *B* or *C*
51. *A* or *C*
52. *R*
53. *K*
54. *G*
55. *Y*
56. *R* or *K* or *G*
57. *R* or *G* or *Y*





58. A recipe calls for $\frac{2}{3}$ cup milk and $\frac{1}{2}$ cup water.
a. What is the total amount of liquid?

b. If you wish to make one-fourth of this recipe, how much of each ingredient is needed, and what is the total amount of liquid?

59. Suppose you have items to mail that weigh $1\frac{1}{4}$ lb, $2\frac{2}{3}$ lb, and $3\frac{1}{2}$ lb. What is the total weight of these packages?

60. Loretta received three boxes of candy for her birthday: a $1\frac{1}{2}$-lb box, a $\frac{3}{4}$-lb box, and a $2\frac{15}{16}$-lb box. What is the total weight of the candy she received?

1.8 Hindu-Arabic Numeration System

The POWER of Math

From Margarita Philosophica Nova, 1523. Courtesy of the Museum of the History of Science, University of Oxford

"Boethius, you old fool!" spat Pythagoras. "Quit being so traditional. Certainly, you must see the benefit of using decimals."

"A—a fool, you say?! You, sir, are a heretic!" retorted Boethius, his eyebrows bobbing about furiously. "You will rot in hell for your beliefs. I've used the numerals of the Roman church all my life, and that will never change."

"My old friend, can't you see Arithmetica in her beautiful robes looking over our disagreement?" asked Pythagoras. "Why do you think she is smiling at me?"

In this section, we will look at the numeration system we use every day. It is called the *Hindu-Arabic* numeration system.

History of Numeration

The system that is in common use today for naming numbers (the one we have been calling the decimal system) has ten symbols—namely, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The selection of ten digits was, no doubt, a result of our having ten fingers (digits).

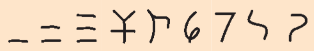
The symbols originated in India about 300 bc. However, because the early specimens do not contain a zero or use a positional system, this numeration offered no advantage over other systems that were then in use in India.

The date of the invention of the zero symbol is not known. The symbol did not originate in India but probably came from the late Babylonian period via the Greek world. By the year a.d. 750, the zero symbol and the idea of a positional system had been brought to Baghdad and translated into Arabic. We are not certain how these numerals were introduced into Europe, but it is likely that they came via Spain in the 8th century. Gerbert, who later became Pope Sylvester II, studied in Spain and was the first European scholar known to have taught these numerals. Because of the origins, these numerals are called the **Hindu-Arabic numerals**. Since ten basic symbols are used, the Hindu-Arabic numeration system is also called the *decimal numeration system*, from the Latin word *decem*, meaning "ten."

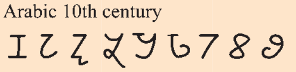
At an early age, we learn our numbers as we learn to count, but if we are asked to define what we mean by *number*, we are generally at a loss. There are many different kinds of numbers, and one type of number is usually defined in terms of more primitive types of numbers. The word *number* is taken as one of our primitive (or undefined) words, but it is used to answer the question "How many?" For example, if we asked how many stars are in this list: ★★★ ★★, you would answer, "Five," but someone else might answer, "Cinq," or "Cinco," or "Fünf." Someone else might respond by saying, "There are '3 + 2' stars." In other words, there is one number 5, but there might be many symbols used to represent the idea of "five." The concept of "fiveness" is called a **number**; the symbol used to represent the concept is called a **numeral**.

Historical Note

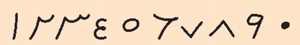
Hindu 300 B.C.



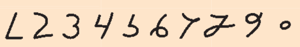
Arabic 10th century



Arabic 15th century



European 15th century



20th century typewriter

1 2 3 4 5 6 7 8 9 0

20th century bank check

1 2 3 4 5 6 7 8 9 0

The symbols used in the Hindu-Arabic numeration system have changed considerably over the centuries. Though coined the Hindu-Arabic numeration system, the Indian numerals were actually adopted by the Persan mathematicians in India, and passed on to the Arabs further west. The most recent variation is the bar codes that can be read by a computer.

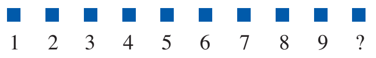
Meaning of Numbers

A **numeration system** consists of a set of basic symbols and some rules for making other symbols from them, the purpose being the identification of all numbers. In the first sections of this book, we have assumed a knowledge of the Hindu-Arabic (decimal) numeration system. The invention of a precise and "workable" way of putting together a set of symbols that is easily learned to represent the multitude of possible numbers is one of the greatest inventions of humanity. It is certainly equal to the invention of the alphabet, which takes 26 letters and uses them to carry the knowledge of one generation to the next.

In this section, we take a deeper look at the Hindu-Arabic numeration system:

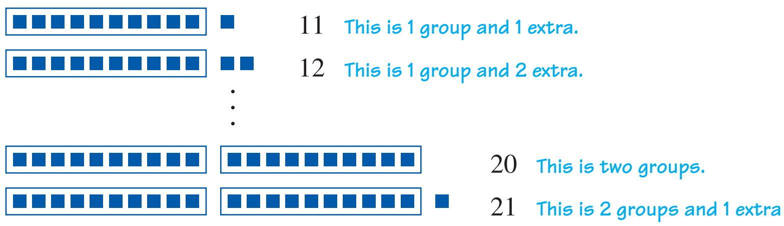
- It uses ten symbols, called digits.
- Larger numbers are expressed in terms of powers of 10.
- It is positional.

Consider how we count objects:

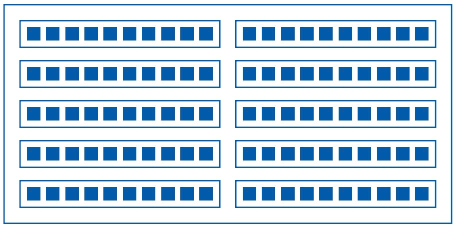


At this point we could invent another symbol as the Egyptians did (you might suggest 0, but remember that 0 represents no objects), or we could reuse the digit symbols by repeating them or by altering their positions. We agree to use 10 to mean 1 group of

We call this group a **ten**. The symbol 0 was invented as a placeholder to show that the 1 here is in a different position from the 1 representing ■. We continue to count:



We continue in the same fashion until we have 9 groups and 9 extra. What's next? It is 10 groups or a group of groups:

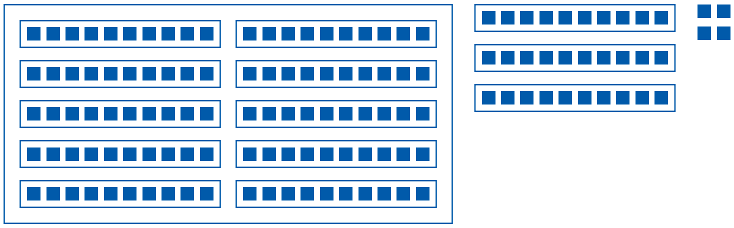


We call this group of groups a $10 \cdot 10$ or 10^2 or a **hundred**. We again use position and repeat the symbol 1 with still different meaning: 100.

EXAMPLE 1 Meaning of a number given in decimal form

What does 134 mean?

Solution **134** means that we have **1** group of 100, **3** groups of 10, and **4** extra. In symbols,



We denote this more simply by writing

These represent the number in each group.

$$(\mathbf{1} \times \underbrace{10^2}) + (\mathbf{3} \times \underbrace{10}) + \mathbf{4}$$

These are the names of the groups.

This leads us to the meaning *one hundred, three tens, four ones.*

Expanded Notation

The representation, or the meaning, of the number 134 in Example 1 is called **expanded notation**.

EXAMPLE 2 Writing a decimal numeral in expanded notation

Write 52,613 in expanded form.

Solution

$$\begin{aligned} 52{,}613 &= 50{,}000 + 2{,}000 + 600 + 10 + 3 \\ &= 5 \times 10^4 + 2 \times 10^3 + 6 \times 10^2 + 1 \times 10 + 3 \end{aligned}$$

EXAMPLE 3 Writing an expanded numeral in decimal notation

Write $4 \times 10^8 + 9 \times 10^7 + 6 \times 10^4 + 3 \times 10 + 7$ in decimal form.

Solution You can use the order of operations and multiply out the digits, b ut you should be able to go directly to decimal form if you remember what place v alue means:

$$4\ 9\ \underbrace{0\ 0\ 6\ 0\ 0}\ 3\ 7 = 490{,}060{,}037$$

Notice that there were no powers of 10^6, 10^5, 10^3, or 10^2.

The positions to the right of the decimal point are fractions:

$$\frac{1}{10} = 10^{-1}, \qquad \frac{1}{100} = 10^{-2}, \qquad \frac{1}{1{,}000} = 10^{-3}$$

To complete the pattern, we also sometimes write $10 = 10^1$ and $1 = 10^0$.

EXAMPLE 4 Writing a decimal numeral with fractional parts in expanded form

Write 479.352 using expanded notation.

Solution

$$\begin{aligned}479.352 &= 400 + 70 + 9 + 0.3 + 0.05 + 0.002\\ &= 400 + 70 + 9 + \frac{3}{10} + \frac{5}{100} + \frac{2}{1{,}000}\\ &= 4 \times 10^2 + 7 \times 10^1 + 9 \times 10^0 + 3 \times 10^{-1} + 5 \times 10^{-2} + 2 \times 10^{-3}\end{aligned}$$

Problem Set 1.8

The POWER of Math

1. IN YOUR OWN WORDS What do we mean by a decimal numeration system?
2. IN YOUR OWN WORDS What is a Hindu-Arabic numeral?
3. IN YOUR OWN WORDS What does this story have to do with your life?

   Hint: The argument is about using Hindu-Arabic numerals or Roman numerals. The Hindu-Arabic numerals were from Islam and were thought of, at the time, as "bad" numerals, whereas the Roman numerals were from Christianity and were thought of, at the time, as "good" numerals. In the Middle Ages, there was a struggle between Islam and Christianity, and it continues to today.
4. IN YOUR OWN WORDS What do you know about Roman numerals? Discuss the benefits of using the Hindu-Arabic numerals as compared with Roman numerals.

LEVEL 1 Essential Ideas

5. IN YOUR OWN WORDS Discuss the difference between "number" and "numeral."
6. IN YOUR OWN WORDS What is expanded notation?
7. What is a group of groups in:
   a. base 10
   b. base 20
8. What is a group of groups in:
   a. base 2
   b. base 8

LEVEL 2 Drill and Practice

Explain each of the concepts or procedures in Problems 9-12.

9. Illustrate the meaning of 123.
10. Illustrate the meaning of 145.
11. Illustrate the meaning of 1,234 by showing the appropriate groupings.
12. Illustrate the meaning of 1,326 by showing the appropriate groupings.

Give the meaning of the numeral 5 in each of the numbers in Problems 13-18.

13. 805
14. 508
15. 0.00567
16. 0.00765
17. 5×10^4
18. 58,000,000

Write the numbers in Problems 19-30 *in decimal notation.*

19. **a.** 10^5
    b. 10^3
20. **a.** 10^{-2}
    b. 10^{-6}
21. **a.** 5×10^3
    b. 5×10^2
22. **a.** 8×10^{-4}
    b. 7×10^{-3}
23. **a.** 6×10^{-2}
    b. 9×10^{-5}

24. a. 5×10^{-6}

b. 2×10^{-9}

25. a. $1 \times 10^4 + 0 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$

b. $6 \times 10^1 + 5 \times 10^0 + 0 \times 10^{-1} + 8 \times 10^{-2} + 9 \times 10^{-3}$

26. a. $5 \times 10^5 + 2 \times 10^4 + 1 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 8 \times 10^0$

b. $6 \times 10^7 + 4 \times 10^3 + 1 \times 10^0$

27. a. $7 \times 10^6 + 3 \times 10^{-2}$

b. $6 \times 10^9 + 2 \times 10^{-3}$

28. $5 \times 10^5 + 4 \times 10^2 + 5 \times 10^1 + 7 \times 10^0 + 3 \times 10^{-1} + 4 \times 10^{-2}$

29. $3 \times 10^3 + 2 \times 10^1 + 8 \times 10^0 + 5 \times 10^{-1} + 4 \times 10^{-2} + 2 \times 10^{-4}$

30. $2 \times 10^4 + 6 \times 10^2 + 4 \times 10^{-1} + 7 \times 10^{-3} + 6 \times 10^{-4} + 9 \times 10^{-5}$

Write each of the numbers in Problems 31-38 in expanded notation.

31. a. 741

b. 728,407

32. a. 0.096421

b. 27.572

33. a. 47.00215

b. 521

34. a. 6,245

b. 2,305,681

35. a. 428.31

b. 5,245.5

36. a. 0.00000527

b. 100,000.001

37. a. 893.0001

b. 8.00005

38. a. 678,000.01

b. 57,285.9361

LEVEL 2 Applications

One of the oldest devices used for calculation is the ***abacus****, as shown in Figure 1.3. Each rod names one of the positions we use in counting. Each bead on the bottom represents one unit in that column, and each bead on the top represents five units in that column. The number illustrated in Figure 1.3 is 1,734. What number is illustrated by the drawings in Problems 39-44?*

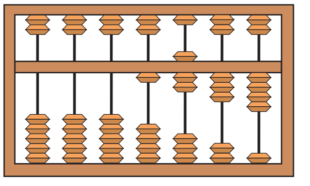
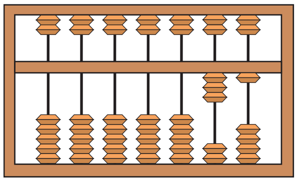


Figure 1.3 Abacus

39.



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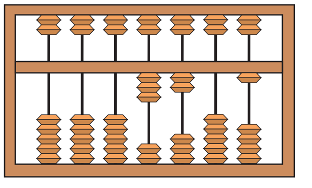
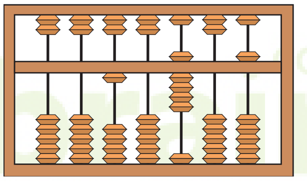
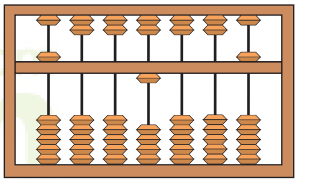


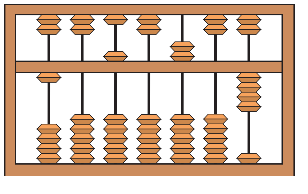
41.



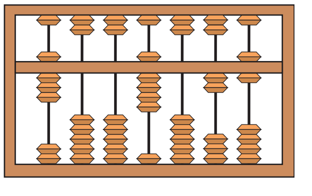
42.



43.



44.



Sketch an abacus to show each of the numbers given in Problems 45-52.

45. 132

46. 849

47. 3,214

48. 9,387

49. 1,998

50. 2,001

51. 3,000,400

52. 8,007,009

Some applications do not lend themselves to the decimal numeration system. Problems 53-60 *are of this type, and we will see in the following section that the use of numeration systems other than the decimal system might be appropriate.*

53. Add 5 years, 7 months to 6 years, 8 months.

54. Add 3 years, 10 months to 2 years, 5 months.

55. Add 10 ft, 7 in. to 7 ft, 10 in.

56. Add 6 ft, 8 in. to 9 ft, 5 in.

57. Add 2 gross, 3 dozen, 4 units to 5 gross, 9 dozen, 10 units

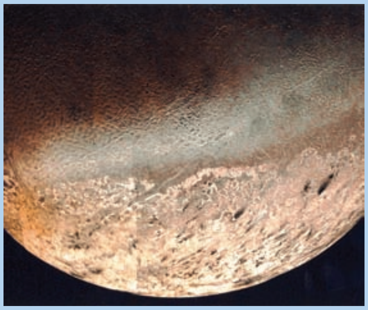
58. Add 1 gross, 9 dozen, 7 units to 2 gross, 8 dozen, 8 units.

59. If you keep one investment for 1 year, 7 months, 11 days, and then roll it over into another investment for 1 year, 6 months, 26 days, how long was the total investment (if you assume that all months are 30 days long)?

60. If you keep one investment for 3 years, 4 months, 21 days, and then roll it over into another investment for 1 year, 6 months, 15 days, how long was the total investment (if you assume that all months are 30 days long)?

1.9 Different Numeration Systems

The POWER of Math



NASA

"Well, my professor stumped me today," admitted George begrudgingly. "He wanted me to add 1 + 1 in something called 'base 2,' whatever that is. He was staring at me, so I just sarcastically said, '2.' And THEN he has the nerve to announce that I was wrong, and the answer was 10!"

"He was right. In base 2, 1 + 1 = 10," said Paul.

"But why would we ever use a base other than base 10?" exclaimed George.

"Do you remember those photos we just saw of Neptune's largest moon? They were sent using a binary numeration system. Trust me, to some people, base 2 literally means the universe."

In the previous section, we discussed the Hindu-Arabic numeration system and grouping by tens. However, we could group by twos, fives, twelves, or any other counting number. In this section, we summarize numeration systems with bases other than ten. This not only will help you understand our own numeration system, but also will give you insight into the numeration systems used with computers, namely, base 2 (**binary**), base 8 (**octal**), and base 16 (**hexadecimal**).

Number Of Symbols

The number of symbols used in a particular base depends on the method of grouping for that base. For example, in base ten the grouping is by tens, and in base five the grouping is by fives. Suppose we wish to count

Historical Note

Karl Smith Library

Native American

The numeration system that we use every day, the one being presented in this chapter, is called the Hindu-Arabic numeration system because it had its origins in India and was brought to Baghdad and translated into Arabic (by the year A.D. 750). It is also sometimes called the **decimal numeration system** because it is based on ten symbols or, as they are sometimes called, digits. A brief discussion of the Hindu-Arabic numeration system is found in Section 1.8.

A study of the Native Americans of California yields a wide variety of number bases different from base 10. Several of these bases are discussed by Barnabas Hughes in an article in the *Bulletin of the California Mathematics Council* (Winter 1971/1972). According to Professor Hughes, the Yukis, who lived north of Willits, had both the quaternary (base 4) and the octal (base 8) systems. Instead of counting on their fingers, they enumerated the spaces between their fingers.

in various bases. Let's look for patterns in Table 1.1. Note the use of the subscript following the numeral to keep track of the base in which we are working.

TABLE 1.1 Grouping in Various Bases

Base	Symbols	Method of Grouping	Notation
2	0, 1	[[(■■)(■■)][(■■)(■■)]] (■■) ■	1011_{two}
3	0, 1, 2	[[■■■][■■■][■■■]] ■■	102_{three}
4	0, 1, 2, 3	[■■■■][■■■■] ■■■	23_{four}
5	0, 1, 2, 3, 4	[■■■■■][■■■■■] ■	21_{five}
6	0, 1, 2, 3, 4, 5	[■■■■■■] ■■■■■	15_{six}
7	0, 1, 2, 3, 4, 5, 6	[■■■■■■■] ■■■■	14_{seven}
8	0, 1, 2, 3, 4, 5, 6, 7	[■■■■■■■■] ■■■	13_{eight}
9	0, 1, 2, 3, 4, 5, 6, 7, 8	[■■■■■■■■■] ■■	12_{nine}
10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	[■■■■■■■■■■] ■	11_{ten}

Do you see any patterns? Suppose we wish to continue this pattern. Can we group by elevens or twelves? We can, provided that new symbols are "invented." For base eleven (or higher bases), we use the symbol T to represent ■■■■■■■■■■. For base twelve (or higher bases), we use E to stand for ■■■■■■■■■■■. For bases larger than twelve, other symbols can be invented.

For example, $2T_{twelve}$ means that there are two groupings of twelve and T (ten) extra:

[■■■■■■■■■■■■] [■■■■■■■■■■■■] ■■■■■■■■■■

We continue with the pattern from Table 1.1 by continuing beyond base ten in Table 1.2.

TABLE 1.2 Grouping in Various Bases

Base	Symbols	Method of Grouping	Notation
11	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T	[■■■■■■■■■■■]	10_{eleven}
12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E	■■■■■■■■■■■	E_{twelve}
13	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E, U	■■■■■■■■■■■	$E_{thirteen}$
14	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E, U, V	■■■■■■■■■■■	$E_{fourteen}$

Do you see more patterns? Can you determine the number of symbols in each base system?

Change from Some Base to Base Ten

To change from base b to base ten, we write the numerals in expanded notation. The resulting number is in base ten.

EXAMPLE 1 Changing to base 10

Change each number to base 10. **a.** 1011.01_{two} **b.** 1011.01_{four} **c.** 1011.01_{five}

Solution

a. $1011.01_{two} = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2}$

$= 8 + 0 + 2 + 1 + 0 + 0.25$

$= 11.25$

b. $1011.01_{four} = 1 \times 4^3 + 0 \times 4^2 + 1 \times 4^1 + 1 \times 4^0 + 0 \times 4^{-1} + 1 \times 4^{-2}$
$= 64 + 0 + 4 + 1 + 0 + 0.0625$
$= 69.0625$

c. $1011.01_{five} = 1 \times 5^3 + 0 \times 5^2 + 1 \times 5^1 + 1 \times 5^0 + 0 \times 5^{-1} + 1 \times 5^{-2}$
$= 125 + 0 + 5 + 1 + 0 + 0.04$
$= 131.04$

Change from Base 10 to Some Base

To see how to change from base 10 to any other valid base, let's again look for a pattern:

To change from base 10 to base 2, group by twos.
To change from base 10 to base 3, group by threes.
To change from base 10 to base 4, group by fours.
To change from base 10 to base 5, group by fives.
⋮

The groupings from this pattern are summarized in the **place-value chart** in Table 1.3.

TABLE 1.3 Place-Value Chart

Base	Place Value					
2	$2^5 = 32$	$2^4 = 16$	$2^3 =$	8 $2^2 =$	4 $2^1 =$	2 $2^0 = 1$
3	$3^5 = 243$	$3^4 = 81$	$3^3 = 27$	$3^2 =$	9 $3^1 =$	3 $3^0 = 1$
4	$4^5 = 1{,}024$	$4^4 = 256$	$4^3 = 64$	$4^2 = 16$	$4^1 =$	4 $4^0 = 1$
5	$5^5 = 3{,}125$	$5^4 = 625$	$5^3 = 125$	$5^2 = 25$	$5^1 =$	5 $5^0 = 1$
6	$6^5 = 7{,}776$	$6^4 = 1{,}296$	$6^3 = 216$	$6^2 = 36$	$6^1 =$	6 $6^0 = 1$
7	$7^5 = 16{,}807$	$7^4 = 2{,}401$	$7^3 = 343$	$7^2 = 49$	$7^1 =$	7 $7^0 = 1$
	8 $^5 = 32{,}768$	$8^4 = 4{,}096$	$8^3 = 512$	$8^2 = 64$	$8^1 =$	8 $8^0 = 1$
10	$10^5 = 100{,}000$	$10^4 = 10{,}000$	$10^3 = 1{,}000$	$10^2 = 100$	$10^1 = 10$	$10^0 = 1$
12	$12^5 = 248{,}832$	$12^4 = 20{,}736$	$12^3 = 1{,}728$	$12^2 = 144$	$12^1 = 12$	$12^0 = 1$

The next example shows how we can interpret this grouping process in terms of a simple division.

EXAMPLE 2 Changing to base two

Convert 42 to base two.

Solution Using Table 1.3, we see that the largest power of two smaller than 42 is 2^5, so we begin with $2^5 = 32$:

$$42 = 1 \times 2^5 + 10$$
$$10 = 0 \times 2^4 + 10$$
10 $$= 1 \times 2^3 + 2$$
2 $$= 0 \times 2^2 + 2$$
2 $$= 1 \times 2^1 + 0$$
0 $$= 0 \times 2^0$$

We could now write out 42 in expanded notation:

$$42 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 101010_{two}$$

Instead of carrying out the steps by using Table 1.3, we will begin with 42 and carry out repeated division, saving each remainder as we go. We are changing to base 2, so we do repeated division by 2:

$$\begin{array}{rl} 21 & \text{r.}0 \leftarrow \text{Save remainder.} \\ 2\overline{)42} & \end{array}$$

Next we need to divide 21 by 2, but instead of rewriting our work, we work our way up:

$$\begin{array}{rl} 10 & \text{r.}1 \leftarrow \text{Save all remainders.} \\ 2\overline{)21} & \text{r.}0 \leftarrow \text{Save remainder.} \\ 2\overline{)42} & \end{array}$$

Continue by doing repeated division.

Stop when you get a zero here.
↓

$$\begin{array}{rl} 0 & \text{r. }1 \quad \text{Answer is found by reading down.} \\ 2\overline{)\ 1} & \text{r. }0 \\ 2\overline{)\ 2} & \text{r. }1 \\ 2\overline{)\ 5} & \text{r. }0 \\ 2\overline{)10} & \text{r. }1 \\ 2\overline{)21} & \text{r. }0 \\ 2\overline{)42} & \end{array}$$

Thus, $42 = 101010_{two}$.

You can check by using expanded notation:

$$101010_{two} = 1 \times 2^5 + 1 \times 2^3 + 1 \times 2 = 32 + 8 + 2 = 42$$

EXAMPLE 3 Changing from base ten to other bases

Write 42 in: **a.** base three **b.** base four

Solution

a. Begin with $3^3 = 27$ (from Table 1.3):

$$\begin{array}{ll} 42 & = 1 \times 3^3 + 15 \\ 15 & = 1 \times 3^2 + 6 \\ 6 & = 2 \times 3^1 + 0 \\ 0 & = 0 \times 3^0 \end{array} \quad \text{or} \quad \begin{array}{rl} 0 & \text{r. }1 \\ 3\overline{)\ 1} & \text{r. }1 \\ 3\overline{)\ 4} & \text{r. }2 \\ 3\overline{)14} & \text{r. }0 \\ 3\overline{)42} & \end{array}$$

Thus, $42 = 1120_{three}$.

b. Begin with $4^2 = 16$ (from Table 1.3):

$$\begin{array}{ll} 42 & = 2 \times 4^2 + 10 \\ 10 & = 2 \times 4^1 + 2 \\ 2 & = 2 \times 4^0 \end{array} \quad \text{or} \quad \begin{array}{rl} 0 & \text{r. }2 \\ 4\overline{)\ 2} & \text{r. }2 \\ 4\overline{)10} & \text{r. }2 \\ 4\overline{)42} & \end{array}$$

Thus, $42 = 222_{four}$.

EXAMPLE 4 Applied number base problem

Suppose you need to purchase 1,000 name tags and can buy them by the gross (144), the dozen (12), or individually. The name tags cost \$0.50 each, \$4.80 per dozen, and \$50.40 per gross. How should you order to minimize the cost?

Solution If you purchase 1,000 tags individually, the cost is $\$0.50 \times 1{,}000 = \$500$. This is not the least cost possible, because of the bulk discounts. We must find the maximum number of gross, then find the number of dozens, and then purchase the remainder individually. We will proceed by repeated division by 12, which we recognize as equivalent to changing the number to base twelve. Change 1,000 to base 12:

$$\begin{array}{rl} 0 & \text{r. } 6 \\ 12\overline{)\;\;\;\;6} & \text{r. } 11\text{, or } E \text{ in base twelve} \\ 12\overline{)\;\;83} & \text{r. } 4 \\ 12\overline{)1{,}000} & \end{array}$$

Thus, $1{,}000 = 6E4_{twelve}$ so you must purchase 6 gross, 11 dozen, and 4 individual tags. Let's check: The cost is $6 \times \$50.40 + 11 \times \$4.80 + 4 \times \$0.50 = \$357.20$. As you can see, this is considerably less expensive than purchasing the individual name tags.

Problem Set 1.9

The POWER of Math

1. IN YOUR OWN WORDS Agree or disagree: "Now I've got you, Smith. This story has absolutely nothing to do with me. I won't ever go into space and I don't care about photos from space, and there is never, ever any chance I'll use a base 2 numeration system."

2. IN YOUR OWN WORDS Do you use computers? What is the numeration system used internally in computers? Compare/contrast your answer here with your answer to Problem 1.

3. IN YOUR OWN WORDS Discuss the binary (base 2) numeration system.

4. IN YOUR OWN WORDS The *duodecimal numeration system* refers to the base twelve system, which uses the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *T*, *E*. Historically, a numeration system based on 12 is not new. There were 12 tribes in Israel and 12 Apostles of Christ. In Babylon, 12 was used as a base for the numeration system before it was replaced by 60. In the 18th and 19th centuries, Charles XII of Sweden and Georg Buffon (1707–1788) advocated the adoption of the base 12 system. Even today there is a Duodecimal Society of America that advocates the adoption of this system. According to the Society's literature, no one "who thought long enough—three to 17 minutes—to grasp the central idea of the duodecimal system ever failed to concede its superiority." Study the duodecimal system from 3 to 17 minutes, and comment on whether you agree with the Society's statement.

LEVEL 1 Essential Ideas

5. IN YOUR OWN WORDS Explain the process of changing from base eight to base ten.

6. IN YOUR OWN WORDS Explain the process of changing from base sixteen to base ten.

7. IN YOUR OWN WORDS Explain the process of changing from base ten to base eight.

8. IN YOUR OWN WORDS Explain the process of changing from base ten to base sixteen.

LEVEL 2 Drill and Practice

9. Count the number of people in the indicated base.

a. base ten **b.** base five
c. base three **d.** base eight
e. base two **f.** base nine

10. Count the number of people in the indicated base.

a. base ten **b.** base five
c. base thirteen **d.** base eight
e. base two **f.** base twelve

In Problems 11-20*, write the numbers in expanded notation.*

11. 643_{eight}

12. 5387.9_{twelve}

13. 110111.1001_{two}

14. 5411.1023_{six}

15. 64200051_{eight}

16. 1021.221_{three}

17. 323000.2_{four}

18. 234000_{five}

19. 3.40231_{five}

20. 2033.1_{four}

Change the numbers in Problems 21-32 *to base ten.*

21. 527_{eight}

22. 527_{twelve}

23. $25TE_{twelve}$

24. 1101.11_{two}

25. 431_{five}

26. 65_{eight}

27. 1011.101_{two}

28. 11101000110_{two}

29. 2110_{three}

30. 4312_{five}

31. 537.1_{eight}

32. 3731_{eight}

33. Change 724 to base five.

34. Change 628 to base four.

35. Change 256 to base two.

36. Change 427 to base twelve.

37. Change 412 to base five.

38. Change 615 to base eight.

39. Change 5,133 to base twelve.

40. Change 615 to base two.

41. Change 512 to base two.

42. Change 795 to base seven.

43. Change 52 to base three.

44. Change 4,731 to base twelve.

45. Change 602 to base eight.

46. Change 76 to base four.

LEVEL 2 Applications

*Use number bases to answer the questions given in Problems*47-60.

47. Change 158 hours to days and hours.

48. Change 52 days to weeks and days.

49. Change 39 ounces to pounds and ounces.

50. Change 55 inches to feet and inches.

51. Change $4.59 to quarters, nickels, and pennies.

52. Change 500 to gross, dozens, and units.

53. Suppose you have two quarters, four nickels, and two pennies. Use base five to write a numeral to indicate your financial status.

54. Using only quarters, nickels, and pennies, what is the minimum number of coins needed to make $0.84?

55. Change $8.34 to the smallest number of coins consisting of quarters, nickels, and pennies.

56. A bookstore ordered 9 gross, 5 dozen, and 4 pencils. Write this number in base twelve and in base ten.

57. Change 54 months to years and months.

58. Change 44 days to weeks and days.

59. Change 49 hours to days and hours.

60. Change 29 hours to days and hours.

1.10 Chapter 1 Summary and Review

Take some time getting ready to work the review problems in this section. First, look back at the definition and property boxes. You will maximize your understanding of this chapter by working the problems in this section only after you have studied the material.

Important Terms

Spending some time with the terms and objectives of this chapter will pay dividends in assuring your success.

Numbers refer to sections of this chapter.

Abacus [1.8]
Addition of fractions [1.7]
Approximately equal to symbol [1.3]
Base [1.5]
Binary [1.9]
Canceling [1.6]
Column names [1.3]
Common denominator [1.7]
Common fraction [1.3]
Completely reduced fraction [1.6]
Complex decimal [1.6]
Composite number [1.5]
Counting number [1.2]
Cubed [1.5]
Decimal [1.3]
Decimal form [1.3]
Decimal fraction [1.3]
Decimal numeration system [1.9]
Decimal point [1.3]
Denominator [1.3]
Difference [1.2]
Distributive property (for multiplication over addition) [1.2]
Divide fractions [1.6]
Division by zero [1.3]
Divisor [1.6]
Elementary operations [1.2]
Estimation [1.2]
Expanded notation [1.8]
Exponent [1.5]
Exponential notation [1.5]
Extended order of operations [1.5]
Factor [1.5]
Factoring [1.5]
Factorization [1.5]
Factor tree [1.5]
Fraction [1.3]
Fundamental property of fractions [1.6]
Googol [1.5]
Hexadecimal [1.9]
Hindu-Arabic numerals [1.8]
Hundred [1.3]
Improper fraction [1.3]
Invert [1.6]
Juxtaposition [1.2]
LCD [1.7]
Lowest common denominator [1.7]
Mixed number [1.3]
Multiply fractions [1.6]
Natural number [1.2]
Number [1.8]
Numeral [1.8]
Numeration system [1.8]
Numerator [1.3]
Numerical expression [1.2]
Octal [1.9]
Order of operations [1.2]
Place-value chart [1.9]
Place-value names [1.3]
Power [1.5]
Powers of ten [1.5]
Prime factorization [1.5]
Prime number [1.5]
Product [1.2]
Proper fraction [1.3]
Quotient [1.2]
Rational number [1.3]
Reciprocal [1.6]
Reduced form [1.6]
Reduced fraction [1.6]
Reducing fractions [1.6]
Remainder [1.3]
Repeating decimal [1.3]
Rounding money [1.4]
Rounding [1.4]
Scientific notation [1.5]
Simplify a fractional expression [1.6]
Simplify a numerical expression [1.2]
Squared [1.5]
Subtraction of fractions [1.7]
Sum [1.2]
Ten [1.3]
Terminating decimal [1.3]
Thousand [1.3]
Trailing zeros [1.3]
Translation [1.2]
Whole numbers [1.2, 1.3]

Learning Outcomes

The material in this chapter is reviewed in the following list of learning outcomes. A self-test (with answers and suggestions for additional study) is given. This self-test is constructed so that each problem number corresponds to a related objective. For example, Problem 7 is testing Objective 1.7. This self-test is followed by a practice test with the questions in mixed order.

[1.1]	*Objective* 1.1	Know some of the symptoms and possible cures for math anxiety.
[1.2]	*Objective* 1.2	Use the order-of-operations agreement to carry out calculations with mixed operations. Classify an expression as a sum, difference, product, or quotient.
[1.2]	*Objective* 1.3	Use the distributive property to eliminate parentheses.

[1.2]	*Objective* 1.4	Translate from words into math symbols.
[1.2–1.7]	*Objective* 1.5	Estimate answers to arithmetic problems.
[1.3]	*Objective* 1.6	Classify a fraction as proper, improper, or a whole number, and be able to enter a fraction into a calculator. Recognize whether the fraction is reduced.
[1.3]	*Objective* 1.7	Write an improper fraction as a mixed number or a whole number.
[1.3]	*Objective* 1.8	Write a mixed number as an improper fraction.
[1.3]	*Objective* 1.9	Change a common fraction to a decimal fraction.
[1.4]	*Objective* 1.10	Round a decimal fraction to a specified degree of accuracy.
[1.5]	*Objective* 1.11	Write a number in scientific notation.
[1.5]	*Objective* 1.12	Write a number without exponents.
[1.5]	*Objective* 1.13	Use the extended order-of-operations agreement.
[1.5]	*Objective* 1.14	Find the prime factorization of a given number.
[1.6]	*Objective* 1.15	Reduce a common fraction.
[1.6]	*Objective* 1.16	Understand the meaning of multiplying and dividing fractions.
[1.6]	*Objective* 1.17	Multiply and divide common fractions.
[1.6]	*Objective* 1.18	Change a decimal fraction to a common fraction.
[1.7]	*Objective* 1.19	Add and subtract common fractions with common denominators.
[1.7]	*Objective* 1.20	Find the LCD for some given denominators.
[1.7]	*Objective* 1.21	Add and subtract common fractions.
[1.7]	*Objective* 1.22	Carry out mixed operations with fractions, including those using juxtaposition.
[1.8]	*Objective* 1.23	Explain (or illustrate) the meaning of a number.
[1.8]	*Objective* 1.24	Write a number in expanded notation as a decimal numeral.
[1.8]	*Objective* 1.25	Write a decimal number in expanded notation.
[1.9]	*Objective* 1.26	Count objects using various bases.
[1.9]	*Objective* 1.27	Write numbers in various bases in expanded notation.
[1.9]	*Objective* 1.28	Change numerals in various bases to base ten.
[1.9]	*Objective* 1.29	Change base ten numerals to a given base.
[1.2–1.9]	*Objective* 1.30	Work applied problems.

Self-Test

Each question of this self-test is related to the corresponding objective listed above.

1. Describe what is meant by math anxiety.

2. Simplify $40 + 20 \div 5 \times 3$, and classify as a sum, difference, product, or quotient.

3. Rewrite $8(500 + 60 + 7)$ using the distributive property.

4. Find (translate) and then simplify:
a. the sum of the squares of three and eleven
b. the square of the sum of three and eleven

5. If you spend $1.85 per day on tolls, estimate the amount you spend each year.

6. Classify each fraction as proper, improper, or a whole number, and then show the keystrokes required to enter each fraction into a calculator. State which of these fractions are reduced.

a. $\frac{5}{2}$ **b.** $\frac{6}{2}$ c. $\frac{2}{5}$ **d.** $\frac{2}{6}$

7. Write $\frac{85}{6}$ as a mixed number.

8. Write $7\frac{3}{8}$ as an improper fraction.

9. Change $6\frac{1}{3}$ to decimal form.

10. Round $85.255 to the nearest cent.

11. Write 93,500,000 in scientific notation.

12. Write 8.92×10^{-9} without exponents.

13. Simplify $4 + 6(8 - 5)^2$.

14. Find the prime factorization of 1,330.

15. Reduce $\frac{1{,}330}{1{,}881}$.

16. **a.** Name the divisor in $18 \div 6$.
b. Find $18 \div 6$ by direct division, and then multiply by the reciprocal.

17. **a.** Simplify: $5\frac{3}{8} \times 2\frac{2}{3}$ **b.** Simplify: $\frac{12}{35} \div \frac{4}{7}$

18. Write $0.8\frac{1}{3}$ as a fraction.

19. Simplify $12\frac{2}{5} - 5\frac{4}{5}$.

20. Find the lowest common denominator for the numbers 120 and 700.

21. Simplify $\frac{3}{10} + \frac{4}{15} + \frac{5}{12}$.

22. Simplify $\frac{2}{3}\left(\frac{3}{8} - \frac{1}{8} \times 2\right)$.

23. Illustrate the meaning of 243.

24. Write $5 \times 10^3 + 6 \times 10^2 + 3 \times 10^{-2}$ in decimal notation.

25. Write 10,063,002 in expanded notation.

26. Count ★★★★★★★★ in

a. base 10 **b.** base 5
c. base 8 **d.** base 2

27. Write 101111.01_{two} in expanded notation.

28. Change $65ET_{twelve}$ to base ten.

29. Change 57 to base two.

30. **a.** A recipe calls for $\frac{2}{3}$ lb of Colombian coffee and $\frac{1}{2}$ lb of another coffee. What is the total weight of the mixture? If you give away one quarter of the mixture, what is the weight of the gift?

b. Change 70 months to years and months. Represent your answer using base 12 notation.

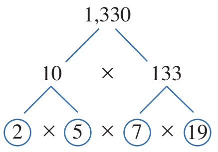
Study Hints for the Self-Test

STOP

Compare your solutions and answers to the self-test. For each problem you missed, work some additional problems in the section listed. After you have worked these problems, you can test yourself with the practice test.

Complete Solutions to the Self-Test	Additional Problems
1. Answers vary: fear; inability to take constructive action; guilt; the more we avoid math, the less able we feel, and the less able we feel, the more we avoid it.	[1.1] Problems 15-26
2. $40 + 20 \div 5 \times 3 = 40 + 4 \times 3$ $= 40 + 12$ $= 52$ *Remember the order of operations.* This is a sum.	[1.2] Problems 11-24; 37-52
3. $8(500 + 60 + 7) = 8(500) + 8(60) + 8(7)$. Use the distributive property; do not carry out the arithmetic.	[1.2] Problems 25-28
4. **a.** $3^2 + 11^2 = 130$ **b.** $(3 + 11)^2 = 196$	[1.2] Problems 29-36
5. $\$1.85 \times 365 \approx \$2 \times 350 = \$700$; you would spend about \$700 per year on tolls. Calculator check: $1.85 \times 365 = 675.25$	[1.2] Problems 53-60; [1.3] Problems 48-51; [1.4] Problems 45-60; [1.5] Problems 46-51; [1.7] Problems 13-20; 46-57
6. **a.** reduced improper fraction $\boxed{5}\ \boxed{\div}\ \boxed{2}$ **b.** whole number $\boxed{6}\ \boxed{\div}\ \boxed{2}$ **c.** reduced proper fraction $\boxed{2}\ \boxed{\div}\ \boxed{5}$ **d.** proper fraction $\boxed{2}\ \boxed{\div}\ \boxed{6}$	[1.3] Problems 14-17
7. $\frac{85}{6} = 14\frac{1}{6}$. Find $85 \div 6 = 14$ with remainder 1.	[1.3] Problems 18-23
8. $7\frac{3}{8} = \frac{59}{8}$. Find $8 \times 7 + 3 = 59$.	[1.3] Problems 24-35
9. $6.\overline{3}$. Do not round your answer; 6.3333333 is not correct; $6.33\frac{1}{3}$ is not a simplified form.	[1.3] Problems 36-47; 52-54
10. \$85.26. The rounding place digit is the hundredth column: $\$85.2\boxed{5}5$.	[1.4] Problems 11-38
11. 9.35×10^7. Count seven decimal places to the right to restore the number to its given form.	[1.5] Problems 31-20
12. 0.00000 00089 2. Count nine decimal places to the left.	[1.5] Problems 21-30
13. $4 + 6(8 - 5)^2 = 4 + 6(3)^2$ $= 4 + 6(9)$ $= 4 + 54$ $= 58$	[1.5] Problems 31-36

14. $2 \times 5 \times 7 \times 19$. Use a factor tree. [1.5] Problems 37-45

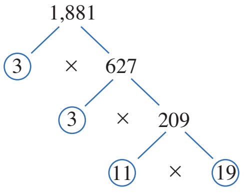


15. $\dfrac{1{,}330}{1{,}881} = \dfrac{2 \times 5 \times 7 \times \cancel{19}}{3 \times 3 \times 11 \times \cancel{19}}$ [1.6] Problems 15-20

$= \dfrac{2 \times 5 \times 7}{3 \times 3 \times 11}$

$= \dfrac{70}{99}$

The factor tree for 1,330 is shown in Problem 14. The other tree is:



16. a. Divisor is 6. [1.6] Problems 21-30

b. $18 \div 6 = 3$; $18 \div 6 = 18 \times \frac{1}{6} = \frac{18}{1} \times \frac{1}{6} = \frac{18}{6} = 3$

Note that the answers are the same.

17. a. $5\frac{3}{8} \times 2\frac{2}{3} = \frac{43}{8} \times \frac{8}{3} = \frac{43}{3}$ You can also write this answer as $14\frac{1}{3}$. [1.6] Problems 31-41

b. $\dfrac{12}{35} \div \dfrac{4}{7} = \dfrac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{35}}} \times \dfrac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{4}}} = \dfrac{3}{5}$

18. $0.8\dfrac{1}{3} = 8\dfrac{1}{3} \times \dfrac{1}{10} = \dfrac{\overset{5}{\cancel{25}}}{3} \times \dfrac{1}{\underset{2}{\cancel{10}}} = \dfrac{5}{6}$ [1.6] Problems 42-49

19. $12\frac{2}{5} - 5\frac{4}{5} = 6\frac{3}{5}$ Write: $12\dfrac{2}{5} = 11\dfrac{7}{5}$ [1.7] Problems 21-23

$$\begin{array}{r} -5\dfrac{4}{5} = 5\dfrac{4}{5} \\ \hline 6\dfrac{3}{5} \end{array}$$

20. $120 = 2^3 \times 3 \times 5$ [1.7] Problems 24-27

$700 = 2^2 \times 5^2 \times 7$

LCD: $2^3 \times 3 \times 5^2 \times 7 = 4{,}200$

21. [1.7] Problems 28-39

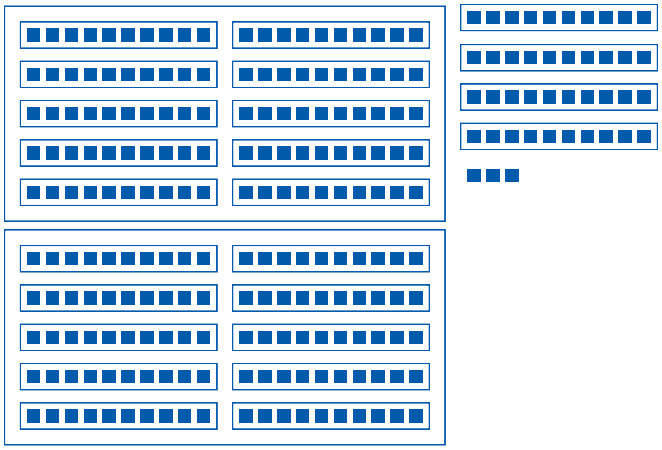
$$\begin{array}{rcl} \dfrac{3}{10} & = & \dfrac{18}{60} \\ \dfrac{4}{15} & = & \dfrac{16}{60} \\ +\dfrac{5}{12} & = & \dfrac{25}{60} \\ \hline & & \dfrac{59}{60} \end{array}$$

22. $\dfrac{2}{3}\left(\dfrac{3}{8} - \dfrac{1}{8} \times 2\right) = \dfrac{2}{3}\left(\dfrac{3}{8} - \dfrac{2}{8}\right)$ Remember order of operations. [1.7] Problems 40-45

$= \dfrac{2}{3}\left(\dfrac{1}{8}\right)$

$= \dfrac{1}{12}$ Parentheses and juxtaposition (no operation symbol) mean multiplication.

23. 243 is 200 + 40 + 3 or 2 hundreds, 4 tens, and 3 units: [1.8] Problems 9-18



24. $5 \times 10^3 + 6 \times 10^2 + 3 \times 10^{-2} = 5{,}000 + 600 + \dfrac{3}{100}$ [1.8] Problems 19-30

$= 5{,}600.03$

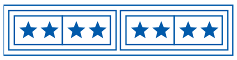
25. $10{,}063{,}002 = 1 \times 10^7 + 0 \times 10^6 + 0 \times 10^5 + 6 \times 10^4 + 3 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 2 \times 10^0$ [1.8] Problems 31-38

$= 1 \times 10^7 + 6 \times 10^4 + 3 \times 10^3 + 2 \times 10^0$

26. a. 8 [1.9] Problems 9-10

b. ; 13_{five}

c. ; 10_{eight}

d. ; 1000_{two}

27. $101111.01_{two} = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2}$ [1.9] Problems 11-20

28. $65\,ET_{twelve} = 6 \times 12^3 + 5 \times 12^2 + 11 \times 12^1 + 10 \times 12^0$ [1.9] Problems 21-32

$= 11{,}230$

29. Use repeated division: [1.9] Problems 33-46

$$\begin{array}{rl}
 0 & \text{r. } 1 \\
2\overline{)\ 1} & \text{r. } 1 \\
2\overline{)\ 3} & \text{r. } 1 \\
2\overline{)\ 7} & \text{r. } 0 \\
2\overline{)14} & \text{r. } 0 \\
2\overline{)28} & \text{r. } 1 \\
2\overline{)57} &
\end{array}$$

The result is found (reading down): 111001_{two}.

Applications

Note: In studying for the exam, be sure you look at several different types of word problems.

30. a. $\dfrac{2}{3} + \dfrac{1}{2} = \dfrac{4}{6} + \dfrac{3}{6} = \dfrac{7}{6}$ The total weight is $\frac{7}{6}$ or $1\frac{1}{6}$ lb.

$\dfrac{1}{4}$ of $\dfrac{7}{6} = \dfrac{1}{4} \times \dfrac{7}{6} = \dfrac{7}{24}$ The size of the gift is $\frac{7}{24}$ lb.

b. Divide 70 by 12 and save the remainder:

$$\begin{array}{rl} 5 & \text{r. } 10 \\ 12\overline{)70} & \end{array}$$

It is 5 years and 10 months, or in base 12: $5T_{twelve}$

[1.1] Problems 37-54
[1.2] Problems 53-60
[1.3] Problems 55-60
[1.4] Problems 39-60
[1.5] Problems 52-60
[1.6] Problems 31-53
[1.7] Problems 46-60
[1.8] Problems 39-60
[1.9] Problems 47-60

Chapter 1 Review Questions

YIELD

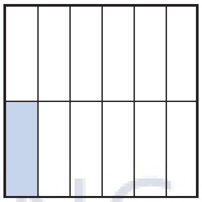
To prepare for a chapter test, first study the chapter; then read each term from the important terms list on page 69, and make sure you know the meaning of each word; finally, review the chapter objectives. ***After*** *these steps, take the self-test, and correct all your answers. The following review questions can be used for extra practice.*

1. **IN YOUR OWN WORDS**

- **a.** List a symptom of math anxiety that you have experienced at some time in your life.
- **b.** How do you think someone with a severe case of math anxiety could learn to deal with this anxiety?
- **c.** Name a math myth that is easy for you to accept as a myth.
- **d.** Name a math myth that is difficult for you not to believe in, even if you know it is a myth.

2. **a.** A box of oranges contains approximately 96 oranges. If the U.S. annual production of oranges is 186,075,000 boxes, estimate the number of oranges produced each year in the United States. Express your answer in scientific notation.

- **b.** Your answer for part **a** is about
  A. 2 million B. 20 million C. 2 trillion
  D. 2 billion E. 20 billion
- **c.** Estimate the portion of the square that is shaded.



- **d.** Illustrate the meaning of 138.

3. Eliminate the parentheses, but do not carry out the arithmetic.

- **a.** $5(8 + 2)$
- **b.** $2(25 + 35)$
- **c.** $3(200 + 50 + 6)$
- **d.** $5(400 + 50 + 9)$

4. **a.** Write the sum of the cubes of 2 and 3.

- **b.** Write the cube of the sum of 2 and 3.
- **c.** Change 13,335 seconds to hours, minutes, and seconds.
- **d.** Use a calculator to estimate 3 trillion dollars divided by the population of the United States, 215 million.

5. Write each as a mixed number.

a. $\frac{114}{7}$ **b.** $\frac{25}{3}$
c. $\frac{167}{10}$ **d.** $\frac{153}{100}$

6. Write each as an improper fraction.

a. $4\frac{2}{3}$ **b.** $1\frac{5}{8}$
c. $3\frac{3}{4}$ **d.** $12\frac{5}{9}$

7. Write in decimal form.

a. $\frac{7}{8}$ **b.** $\frac{5}{6}$
c. $8\frac{2}{3}$ **d.** $2\frac{4}{5}$

8. **a.** Round 6.149 to the nearest tenth.

- **b.** Round 45.5 to the nearest unit.
- **c.** Round \$45.31499 to the nearest dollar.
- **d.** Round \$104.996 to the nearest cent.

9. Write in scientific notation.

- **a.** 0.0034
- **b.** 4,000,300
- **c.** 17,400
- **d.** 5

10. Write without exponents.

- **a.** 4^3
- **b.** 9^2
- **c.** 5.79×10^{-4}
- **d.** 4.01×10^5

11. Write the prime factorization.

a. 86 **b.** 72
c. 486 **d.** 1,372

12. Reduce each fraction.

a. $\frac{15}{25}$ **b.** $\frac{32}{16}$
c. $\frac{192}{240}$ **d.** $\frac{128}{384}$

13. Write in fractional form.

a. 0.333 **b.** $0.2\frac{2}{9}$
c. 0.95 **d.** $0.00\frac{1}{2}$

14. Find the LCD.

a. 12; 15 **b.** 6; 10
c. 10; 15; 6 **d.** 24; 30; 18

15. Simplify and classify each as a sum, difference, product, or quotient.

- **a.** $12 + 20 \div 2$
- **b.** $(12 + 20) \div 2$
- **c.** $8 + 3 \times 6 \div 2$
- **d.** $(8 + 3) - (6 \div 2)$

Simplify the expressions in Problems 16-20.

16. **a.** $\frac{3}{5} \times \frac{25}{27}$ **b.** $\frac{4}{9} \times 27$
c. $4\frac{1}{6} \times 3\frac{2}{5}$ **d.** $2\frac{3}{4} \times \frac{4}{5}$

17. **a.** $\frac{5}{8} \div \frac{1}{2}$ **b.** $\frac{14}{25} \div \frac{7}{5}$
c. $1\frac{1}{2} \div \frac{3}{4}$ **d.** $6\frac{1}{2} \div 3\frac{3}{4}$

18. **a.** $\frac{5}{7} + \frac{3}{7}$ **b.** $\frac{6}{11} - \frac{2}{11}$
c. $12\frac{4}{5} + 6\frac{3}{5}$ **d.** $5\frac{1}{3} - 1\frac{2}{3}$

19. **a.** $\frac{3}{8} + \frac{1}{6}$ **b.** $\frac{7}{12} - \frac{2}{15}$
c. $7\frac{2}{15} - 3\frac{7}{12}$ **d.** $\frac{4}{10} + \frac{7}{15} - \frac{5}{6}$

20. **a.** $\frac{2}{3} + \frac{4}{5} \div \frac{1}{2}$ **b.** $\dfrac{2 \times 8 + 3 \times 5}{3 \times 8}$
c. $\frac{4}{5} \times \frac{12}{23} - \frac{4}{5} \times \frac{2}{23}$ **d.** $\frac{4}{5}\left(\frac{12}{23} - \frac{2}{23}\right)$

21. Rework Problem 20 using a calculator; leave your answer in decimal form. Classify each as a sum, difference, product, or quotient.

22. **a.** Change 11011_{two} to base ten.
b. Change 716_{eight} to base ten.
c. Change one million to base five.
d. Change one million to base two.

23. Write each number in decimal notation.
a. $4 \times 10^3 + 6 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-2}$
b. $4 \times 8^3 + 6 \times 8^1 + 3 \times 8^0 + 2 \times 8^{-2}$
c. $1 \times 2^3 + 1 \times 2^0 + 1 \times 2^{-1}$
d. $3 \times 12^2 + 4 \times 12^1 + 3 \times 12^0$

24. If you join a book club and agree to buy six books at \$24.95 each plus \$3.50 postage and handling for each book, what is the total cost to fulfill this agreement?

25. Enter your favorite number (a counting number from 1 to 9) into a calculator. Multiply by 259; then multiply this result by 429. What is your answer? Try it for three different choices.

Individual Projects

Learning to use sources outside your classroom and textbook is an important skill. Here are some ideas for extending some of the ideas in this chapter.

PROJECT 1.1 **Start a Journal** Create a first entry in your journal. Write down five ideas concerning your commitment to keeping a journal. Tomorrow, write in your journal several reasons for keeping a journal. After that, write in your journal within 24 hours of each math class that you attend.

PROJECT 1.2 In *Everybody Counts,* Lynn Steen states, "Mathematics is alive and constantly changing. As we complete the last decade of this century, we stand on the threshold of major changes in the mathematics curriculum in the United States." Report on some of these recent changes.

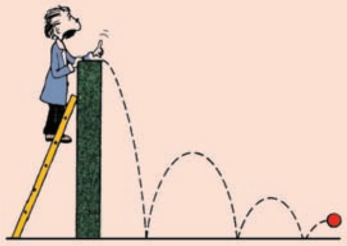
REFERENCES Lynn Steen, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education.* (Washington, DC: National Academy Press, 1989). See also *Curriculum and Evaluation Standards for School Mathematics* from the National Council of Teachers of Mathematics (Reston, VA: NCTM, 1989).

PROJECT 1.3 Insert appropriate operation signs ($+$, $-$, $\times$, or $\div$) between consecutive digits so that the following becomes a true statement:

1 2 3 4 5 6 7 8 9 = 100

PROJECT 1.4 Imagine that you have written the numbers from 1 to 1,000,000. What is the total number of zeros you have recorded?

PROJECT 1.5 A rubber ball is known to rebound half the height it drops.



If the ball is dropped from a height of 100 feet, how far will it have traveled by the time it hits the ground for:

a. The first time?
b. The second time?
c. The third time?
d. The fourth time?
e. The fifth time?
f. Look for a pattern, and decide whether there is a maximum distance the ball will travel if we assume that it will bounce indefinitely.

PROJECT 1.6 Look for a pattern in the following problem. Verify by division (show your work).

a. $\frac{1}{9} = 0.111\ldots$ **b.** $\frac{2}{9} = 0.222\ldots$ **c.** $\frac{3}{9} = 0.333\ldots$ **d.** $\frac{4}{9} = 0.444\ldots$
e. $\frac{8}{9} = 0.888\ldots$ **f.** What is $\frac{9}{9}$ according to the pattern?

PROJECT 1.7 Can you find a pattern?

0, 1, 2, 10, 11, 12, 20, 21, 22, 100, . . .

PROJECT 1.8 Notice the following pattern for multiplication by 11:

$$14 \times 11 = 1\_4 \qquad 51 \quad \times 11 = 5\_1$$

First, copy the first and last digits of the number to be multiplied by 11. Leave a space between these digits. Then insert the sum of the original two digits between those original digits:

$$14 \times 11 = 1\,\mathbf{5}\,4 \qquad 51 \quad \times 11 = 5\,\mathbf{6}\,1$$

$\uparrow\uparrow$

1 $+ 4 = 5$ $5 + 1 = 6$

Use expanded notation to show why this pattern "works."

Team Projects

Working in small groups is typical of most work environments, and learning to work with others to communicate specific ideas is an important skill. Work with three or four other students to submit a single report based on each of the following questions.

T1. Working with others can be beneficial not only on the job, but also in the classroom. For this team project, introduce yourself to three or four classmates, and work with them for this problem. Spend at least 30 minutes getting to know one another, specifically focusing on these statements about your previous mathematics experiences:

"Everybody knows what to do, except me!"
"I got the right answer, but I don't know how!"
"I'm sure I learned it, but I can't remember what to do!"
"This may be a stupid question, but . . . ."
"I'm no good at numbers!"
"Math is unrelated to my life!"
"Math is my worst subject!"
"I don't have a math mind!"

Write a paper summarizing your discussion, and submit one paper for your team.

T2. If it takes 1 second to say each number, how long would it take (to the nearest year) to count to a billion. Assume nonstop counting.

T3. If it takes 1 second to write down each digit, how long would it take to write all the numbers from 1 to 1,000,000?

T4. Inthe *B.C.* cartoon, Peter has a mental block against 4s. See whether you can handle 4s by writing the numbers from 1 to 10 using four 4s for each.

B.C. By permission of John L. Hart ©2011 Creators Syndicate, Inc.

Here are the first three completed for you:

$$\frac{4}{4} + 4 - 4 = 1 \qquad \frac{4}{4} + \frac{4}{4} = 2 \qquad \frac{4+4+4}{4} = 3$$

More than one answer is possible. For example,

$$\frac{4}{4+4-4} = 1 \qquad \text{and} \qquad 4 - \frac{4+4}{4} = 2$$

ANTICIPATE

- **Overview; check out contents, terms, essential ideas, and learning outcomes.**
- **Integers—have you worked with positive and negative numbers before?**
- ***Algebra* is defined as a generalization of arithmetic and has been described as one of the best labor-saving devices invented by the human mind.**
- **Don't go into "symbol shock" when you work in this chapter.**

Kevin Curry/Getty Images

ESSENTIAL IDEAS

To help you succeed in this course, we have included these essential ideas as problems at the beginning of each section.

Section	Essential Ideas	Problems
2.1	Add terms, multiply factors	Problems 3-4
	Translations for operation symbols	Problem 5
	Meaning of a variable	Problem 6
	Evaluate an expression	Problem 7
	Difference of squares/square of a difference	Problem 8
2.2	Set of integers	Problem 3
	Procedure for adding integers	Problems 4-7
	Evaluate an absolute value	Problem 8
2.3	Procedure for subtracting integers	Problem 3
	Three uses of the " – " symbol: minus, negative, and opposite	Problem 4
2.4	Define multiplication	Problem 3
	Opposite property	Problem 4
	Zero multiplication property	Problem 5
	Procedure for multiplying integers	Problems 6 and 7
	Procedure for evaluating an algebraic expression	Problem 8
2.5	Procedure for dividing integers	Problem 3
	Reason why division by zero is impossible	Problem 2
	Procedure for finding the mean of a set of numbers	Problem 4
	Using a fractional bar for division	Problems 5-6
2.6	Perfect squares less than 200	Problem 2
	Reduced fraction	Problem 3
	Standard forms of a reduced fraction	Problem 4
	Pythagorean theorem	Problem 5
	Characterizations of the real numbers	Problem 6
	Meaning of square root	Problems 7-10

APPENDIX B ANSWERS TO SELECTED PROBLEMS

Chapter 1

1.1 Math Anxiety, page 9

Throughout this book, you will find problems that are designated **IN YOUR OWN WORDS.** Since these are opinion questions, the answers are not right or wrong, so we do not show answers to these questions in the answer section. However, that does not mean that any answer is correct. You need to attempt to answer each of these questions honestly, and you need to back up your arguments with examples and facts. One-word (and usually one-sentence) answers will not receive full credit. **3.** This *stop sign* designates important material, and you should stop, understand, and study the highlighted concept. **5.** The *yield sign* is used throughout the book to tell you to move slowly and remember the result. **37.** Stop ahead **39.** Roundabout ahead **41.** Kangaroo crossing **43.** This sign is used to designate the location of a women's rest room. **45.** This sign is used to designate the location of a currency exchange service. **47.** This sign is used to designate the location of a restaurant. **49.** B, employment **51.** A, announcements **53.** E, recreation

1.2 Formulating the Problem, page 18

1. 4 **3.** Parentheses first; then multiplication and division, reading from left to right; finally, addition and subtraction, reading from left to right **5.** F **7.** F **9.** F **11. a.** 17; sum **b.** 14; sum **13. a.** 5; sum **b.** 5; quotient **15. a.** 17; sum **b.** 14; difference **17. a.** 32; sum **b.** 56; sum **19. a.** 27; sum **b.** 13; sum **21. a.** 9; sum **b.** 19; sum **23. a.** 29; difference **b.** 8; difference **25. a.** $3 \times 4 + 3 \times 8$ **b.** $7 \times 9 + 7 \times 4$ **27. a.** $4 \times 300 + 4 \times 20 + 4 \times 7$ **b.** $6 \times 500 + 6 \times 30 + 6 \times 3$ **29.** $3 + 2 \times 4$ **31.** $10(5 + 6)$ **33.** $8 \times 5 + 10$ **35.** $8(11 - 9)$ **37.** 261; difference **39.** 800; sum **41.** 1,080; sum **43.** 1,600; product **45.** 59; difference **47.** 2,700; difference **49.** 285,197; sum **51.** 2,323; sum **53.** 8,640 hours **55.** \$769.83 **57.** \$37,440 **59.** 345 miles

1.3 Fractions and Decimals, page 26

5. The place value names (in decreasing order) are: trillions, hundred billions, ten billions, billions, hundred millions, ten millions, millions, hundred thousands, ten thousands, thousands, hundreds, tens, units, decimal point, tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, and millionths. **7.** T **9.** T **11.** F **13.** T **15. a.** proper; $\boxed{8}\ \boxed{\div}\ \boxed{13}\ \boxed{=}$ **b.** proper; $\boxed{17}\ \boxed{\div}\ \boxed{21}\ \boxed{=}$ **17. a.** $\boxed{4}\ \boxed{+}\ \boxed{3}\ \boxed{\div}\ \boxed{7}\ \boxed{=}$ **b.** $\boxed{2}\ \boxed{+}\ \boxed{1}\ \boxed{\div}\ \boxed{2}\ \boxed{=}$ **19. a.** $5\frac{1}{3}$ **b.** $6\frac{1}{4}$ **c.** $14\frac{1}{10}$ **d.** $16\frac{3}{10}$ **21. a.** $1\frac{11}{16}$ **b.** $17\frac{7}{10}$ **c.** $2\frac{1}{16}$ **d.** $3\frac{6}{7}$ **23. a.** $16\frac{3}{5}$ **b.** 14 **c.** 6 **d.** $31\frac{1}{4}$ **25. a.** $\frac{7}{4}$ **b.** $\frac{18}{5}$ **27. a.** $\frac{11}{3}$ **b.** $\frac{26}{5}$ **29. a.** $\frac{13}{10}$ **b.** $\frac{22}{5}$ **31. a.** $\frac{53}{3}$ **b.** $\frac{64}{5}$ **33. a.** $\frac{29}{15}$ **b.** $\frac{47}{17}$ **35. a.** $\frac{151}{8}$ **b.** $\frac{47}{12}$ **37. a.** $0.8\overline{3}$ **b.** $1.1\overline{6}$ **39. a.** 2.5 **b.** $5.\overline{3}$ **41. a.** $6.08\overline{3}$ **b.** $6.\overline{6}$ **43. a.** $7.8\overline{3}$ **b.** $4.\overline{06}$ **45. a.** $2.\overline{6}$ **b.** $4.1\overline{6}$ **47. a.** 4.375 **b.** $3.\overline{1}$ **49. a.** $\frac{41}{50}$ **b.** $\frac{5}{8}$ **51. a.** 0.63 **b.** 0.82 **53. a.** $0.4\overline{6}$ **b.** $0.\overline{318}$ **55.** \$5,250 **57.** \$8,450 **59.** \$6,312.50

1.4 Rounding and Estimation, page 31

5. F **7.** F **9.** T **11.** 2.3 **13.** 6,287.45 **15.** 5.3 **17.** 6,300 **19.** 12.82 **21.** 4.818 **23.** 5 **25.** \$12.99 **27.** \$15.00 **29.** 690 **31.** \$86,000 **33.** 0.667 **35.** 0.118 **37.** 0.137 **39.** 0.333 **41.** 0.417 **43.** 0.318 **45.** B **47.** C **49.** B **51.** \$1,250 **53.** \$12.13 **55.** \$70.83 **57.** \$112.33

1.5 Exponents and Prime Factorization, page 39

7. a. one million **b.** 10 **c.** 6 **d.** $10 \times 10 \times 10 \times 10 \times 10 \times 10$ **9. a.** one-tenth **b.** 10 **c.** -1 **d.** 0.1 **11.** F **13.** F **15. a.** 3.2×10^3 **b.** 2.5×10^4 **c.** 1.8×10^7 **d.** 6.4×10^2 **17. a.** 4.21×10^{-6} **b.** 9.2×10^7 **c.** 1 or 10^0 **d.** 1.5×10^0 **19. a.** 6.34×10^9 **b.** 5.2019×10^{11} **c.** 4.093745×10^8 **d.** $8.291029292 \times 10^{12}$ **21. a.** 72,000,000,000 **b.** 4,500 **23. a.** 0.0021 **b.** 0.00000 046 **25. a.** 3.2 **b.** 0.00080 3 **27. a.** 49 **b.** 25 **29. a.** 21,892,827,100 **b.** 329 **31.** 43 **33.** 33 **35.** 169 **37. a.** $2^2 \times 3$ **b.** $2^2 \times 5$ **39. a.** 2^8 **b.** 2×3^2 **41. a.** $2^4 \times 5^2$ **b.** $2^3 \times 5^3$ **43. a.** $7^3 \times 13$ **b.** $13^2 \times 23 \times 59$ **45. a.** $19 \times 29 \times 83$ **b.** 31^3 **47.** A **49.** C **51.** A **53.** 4.184×10^7 **55.** 333,000 **57. a.** 500,000,000,000,000,000,000,000 **b.** 1.5768×10^{31} **59.** \$20

1.6 Common Fractions, page 48

9. T **11.** F **13.** F **15. a.** $\frac{1}{2}$ **b.** $\frac{1}{3}$ **c.** $\frac{1}{4}$ **d.** $\frac{1}{5}$ **17. a.** $\frac{24}{5}$ **b.** 3 **c.** $\frac{2}{3}$ **d.** $\frac{1}{2}$ **19. a.** $\frac{1}{3}$ **b.** $\frac{3}{2}$ **c.** $\frac{5}{14}$ **d.** $\frac{7}{15}$ **21.** Shade 6 of the 20 squares. **23.** Shade 4 of the 15 squares. **25.** Shade 2 of the 18 squares. **27. a.** divisor, 5; 3; 3 **b.** divisor, 3; 2; 2 **29. a.** divisor, 4; 5; 5 **b.** divisor, 16; 15; 15 **31. a.** $\frac{1}{24}$ **b.** $\frac{8}{15}$ **c.** $\frac{1}{8}$ **d.** $\frac{5}{2}$ **e.** $\frac{2}{5}$ **f.** $\frac{1}{4}$ **33. a.** $\frac{3}{2}$ **b.** $\frac{2}{3}$ **c.** $\frac{4}{3}$ **d.** $\frac{9}{8}$ **e.** $\frac{3}{4}$ **f.** 1 **35. a.** 1 **b.** 1 **c.** 1 **d.** 4 **e.** $\frac{7}{5}$ **f.** $\frac{9}{7}$ **37. a.** $\frac{2}{15}$ **b.** $\frac{1}{8}$ **c.** $\frac{3}{25}$ **d.** 18 **e.** $\frac{5}{6}$ **f.** $\frac{13}{6}$ **39. a.** $\frac{108}{25}$ **b.** $\frac{121}{6}$ **c.** $\frac{225}{16}$ **d.** $\frac{49}{18}$ **e.** $\frac{65}{12}$ **f.** $\frac{19}{10}$ **41. a.** $\frac{1}{12}$ **b.** $\frac{1}{15}$ **c.** $\frac{1}{2}$ **d.** $\frac{19}{2}$ **e.** 4 **f.** $\frac{3}{8}$ **43. a.** $\frac{1}{4}$ **b.** $\frac{87}{100}$ **c.** $\frac{3}{8}$ **45. a.** $\frac{39}{50}$ **b.** $\frac{17}{20}$ **c.** $\frac{123}{500}$ **47. a.** $\frac{2}{3}$ **b.** $\frac{7}{8}$ **c.** $\frac{1}{6}$ **49. a.** $\frac{1}{9}$ **b.** $\frac{5}{9}$ **c.** $\frac{1}{12}$ **51.** \$138,000 **53.** \$625,000 **55.** \$2,346 **57.** \$11.35 **59.** \$10,000

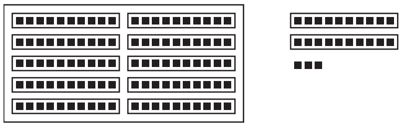
1.7 Adding and Subtracting Fractions, page 56

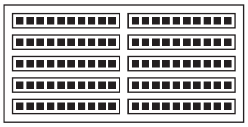
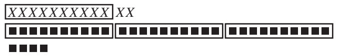
3. $18\frac{1}{16}$ in. **7.** F **9.** T **11.** T **13.** A **15.** D **17.** B **19.** A **21. a.** $\frac{3}{5}$ **b.** $\frac{8}{7}$ **c.** $\frac{8}{11}$ **d.** 4 **e.** 1 **f.** $\frac{2}{3}$ **23. a.** 4 **b.** 9 **c.** 13 **d.** $1\frac{2}{3}$ **e.** $5\frac{1}{2}$ **f.** $\frac{1}{2}$ **25. a.** 12 **b.** 180 **c.** 336 **d.** 630 **27. a.** 55,125 **b.** 2,205 **c.** 6,300 **d.** 1,800 **29. a.** $\frac{1}{2}$ **b.** $\frac{35}{24}$ **c.** $\frac{7}{24}$ **d.** $\frac{1}{3}$ **e.** $\frac{23}{30}$ **f.** $\frac{5}{6}$ **31. a.** $7\frac{1}{4}$ **b.** $7\frac{1}{6}$ **c.** $8\frac{7}{8}$ **33. a.** $2\frac{23}{24}$ **b.** $1\frac{37}{70}$ **35. a.** $25\frac{1}{3}$ **b.** 20 **37. a.** $\frac{8}{15}$ **b.** $\frac{12}{5}$ **39. a.** $\frac{19}{15}$ **b.** $\frac{43}{35}$ **41.** 0.8 **43.** $9.\overline{3}$ **45.** 0.4487116145 (approx.) **47.** $\frac{7}{24}$ **49.** $\frac{7}{12}$ **51.** $\frac{17}{24}$ **53.** $\frac{1}{6}$ **55.** $\frac{5}{18}$ **57.** $\frac{5}{6}$ **59.** $7\frac{5}{12}$ pounds

1.8 Hindu-Arabic Numeration System, page 61

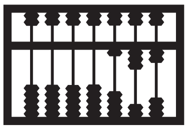
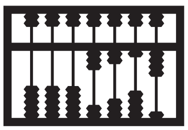
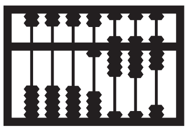
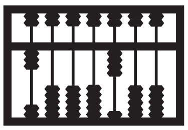
7. a. 100 **b.** 400

9.



11. Let X = . Then 1,234 can be represented as 

13. 5 units **15.** 5 thousandths **17.** 5 ten thousands **19. a.** 100,000 **b.** 1,000 **21. a.** 5,000 **b.** 500 **23. a.** 0.06 **b.** 0.00009 **25. a.** 10,234 **b.** 65.089 **27. a.** 7,000,000.03 **b.** 6,000,000,000.002 **29.** 3,028.5402 **31. a.** $7 \times 10^2 + 4 \times 10 + 1$ **b.** $7 \times 10^5 + 2 \times 10^4 + 8 \times 10^3 + 4 \times 10^2 + 7$ **33. a.** $4 \times 10^1 + 7 + 2 \times 10^{-3} + 1 \times 10^{-4} + 5 \times 10^{-5}$ **b.** $5 \times 10^2 + 2 \times 10 + 1$ **35. a.** $4 \times 10^2 + 2 \times 10^1 + 8 + 3 \times 10^{-1} + 1 \times 10^{-2}$ **b.** $5 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 5 + 5 \times 10^{-1}$ **37. a.** $8 \times 10^2 + 9 \times 10^1 + 3 + 1 \times 10^{-4}$ **b.** $8 + 5 \times 10^{-5}$ **39.** 31 **41.** 10,905 **43.** 1,051,004

45.  **47.**  **49.**  **51.** 

53. 12 years, 3 months **55.** 18 ft, 5 in. **57.** 8 gross, 1 dozen, 2 units **59.** 3 years, 2 months, 7 days

1.9 Different Numeration Systems, page 67

9. a. 9 **b.** 14_{five} **c.** 100_{three} **d.** 11_{eight} **e.** 1001_{two} **f.** 10_{nine} **11.** $6 \times 8^2 + 4 \times 8^1 + 3 \times 8^0$ **13.** $1 \times 2^5 + 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-4}$ **15.** $6 \times 8^7 + 4 \times 8^6 + 2 \times 8^5 + 5 \times 8^1 + 1 \times 8^0$ **17.** $3 \times 4^5 + 2 \times 4^4 + 3 \times 4^3 + 2 \times 4^{-1}$ **19.** $3 \times 5^0 + 4 \times 5^{-1} + 2 \times 5^{-3} + 3 \times 5^{-4} + 1 \times 5^{-5}$ **21.** 343 **23.** 4,307 **25.** 116 **27.** 11.625 **29.** 66 **31.** 351.125 **33.** 10344_{five} **35.** 100000000_{two} **37.** 3122_{five} **39.** $2E79_{twelve}$ **41.** 1000000000_{two} **43.** 1221_{three} **45.** 1132_{eight} **47.** 6 days, 14 hours **49.** 2 lb, 7 oz **51.** 18 quarters, 1 nickel, 4 pennies **53.** 242_{five}; financial status is \$0.72 **55.** 33 quarters, 1 nickel, 4 pennies **57.** $54 = 46_{twelve}$; 4 years, 6 months **59.** $49 = 21_{twenty\text{-}four}$; 2 days, 1 hour

Chapter 1 Review Questions, page 74

3. a. $5 \times 8 + 5 \times 2$ **b.** $2 \times 25 + 2 \times 35$ **c.** $3 \times 200 + 3 \times 50 + 3 \times 6$ **d.** $5 \times 400 + 5 \times 50 + 5 \times 9$ **5. a.** $16\frac{2}{7}$ **b.** $8\frac{1}{3}$ **c.** $16\frac{7}{10}$ **d.** $1\frac{53}{100}$ **7. a.** 0.875 **b.** $0.8\overline{3}$ **c.** $8.\overline{6}$ **d.** 2.8 **9. a.** 3.4×10^{-3} **b.** 4.0003×10^6 **c.** 1.74×10^4 **d.** 5 **11. a.** 2×43 **b.** $2^3 \times 3^2$ **c.** 2×3^5 **d.** $2^2 \times 7^3$ **13. a.** $\frac{333}{1,000}$ **b.** $\frac{2}{9}$ **c.** $\frac{19}{20}$ **d.** $\frac{1}{200}$ **15. a.** 22; sum **b.** 16; quotient **c.** 17; sum **d.** 8; difference **17. a.** $\frac{5}{4}$ **b.** $\frac{2}{5}$ **c.** 2 **d.** $\frac{26}{15}$ **19. a.** $\frac{13}{24}$ **b.** $\frac{9}{20}$ **c.** $3\frac{11}{20}$ **d.** $\frac{1}{30}$ **21. a.** 2.266666667; sum **b.** 1.291666667; quotient **c.** 0.347826087; difference **d.** 0.347826087; product **23. a.** 4,063.02 **b.** 2,099.03125 **c.** 9.5 **d.** 483 **25.** Your favorite digit is repeated six times.