

HSD Elementary Mathematics Learning Progressions by Domain

Number Sense

Counting & Cardinality is about understanding and using numbers. Counting & Cardinality underlies Number Sense and Operations in Base Ten. It begins with early counting and telling how many in one group of objects. Addition, subtraction, multiplication, and division grow from these early roots. From its very beginnings, this Progression involves important ideas that are neither trivial nor obvious; these ideas need to be taught, in ways that are interesting and engaging to young students.

Kindergarten	1 st Grade
Know the number names and the count sequence.	Understand and use numbers up to 120.
<ul style="list-style-type: none"> Count to 100 by ones and tens. Count forward beginning from a given number between 1 and 20. Count backward from a given number between 10 and 1. Read and write numerals and represent a number of objects from 0 to 20. 	<ul style="list-style-type: none"> Count to 120, starting at any number less than 120. Read and write numerals and represent a number of objects with a written numeral. Count backward from a given number between 20 and 1. Count by 5s to 100 starting at any multiple of five.
Understand the relationship between numbers and quantities; connect counting to cardinality.	
<ul style="list-style-type: none"> Say the number names when counting objects, in the standard order, pairing each object with one and only one number name and each number name with one and only one object. Demonstrate that the last number name said tells the number of objects counted and the number of objects is the same regardless of their arrangement or the order in which they were counted. Demonstrate that each successive number name refers to a quantity that is one larger than the previous number. Recognize, without counting, the quantity of groups up to 5 objects arranged in common patterns. Demonstrate that a number can be used to represent “how many” are in a set. 	
Compare numbers.	
<ul style="list-style-type: none"> Compare two or more sets of objects and identify which set is equal to, more than or less than the other. Compare two numerals, between 1 and 10, and determine which is more than or less than the other. 	

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Number Sense and Operations in Base Ten

<p>Students' work in the base-ten system is intertwined with their work on counting and cardinality, and with the meanings and properties of addition, subtraction, multiplication, and division. Work in the base-ten system relies on these meanings and properties, but also contributes to deepening students' understanding of them.</p>				
Kindergarten	2 nd Grade	3 rd Grade	4 th Grade	5 th Grade
<p>Work with numbers 11-19 to gain foundations for place value.</p>	<p>Understand place value of three digit numbers.</p>	<p>Use place value understanding and properties of operations to perform multi-digit arithmetic.</p>	<p>Use place value understanding and properties of operations to perform multi-digit arithmetic with numbers up to one million.</p>	<p>Use place value system understanding to perform operations with multi-digit whole numbers to billions and decimals to thousandths.</p>
<ul style="list-style-type: none"> • Compose and decompose numbers from 11 to 19 into sets of tens with additional ones. 	<ul style="list-style-type: none"> • Understand three-digit numbers are composed of hundreds, tens and ones. • Understand that 100 can be thought of as 10 tens – called a “hundred”. • Count within 1000 by 1s, 10s and 100s starting with any number. • Read and write numbers to 1000 using number names, base-ten numerals and expanded form. • Compare two three-digit numbers using the symbols $>$, $=$ or $<$. 	<ul style="list-style-type: none"> • Round whole numbers to the nearest 10 or 100. • Read, write and identify whole numbers within 100,000 using base ten numerals, number names and expanded form • Demonstrate fluency with addition and subtraction within 1000. • Multiply whole numbers by multiples of 10 in the range 10-90. 	<ul style="list-style-type: none"> • Round multi-digit whole numbers to any place. • Read, write and identify multi-digit whole numbers up to one million using number names, base ten numerals and expanded form. • Compare two multi-digit numbers using the symbols $>$, $=$ or $<$, and justify the solution. • Understand that in a multi-digit whole number, a digit represents 10 times what it would represent in the place to its right. • Demonstrate fluency with addition and subtraction of whole numbers. 	<ul style="list-style-type: none"> • Read, write and identify numbers from billions to thousandths using number names, base ten numerals and expanded form. • Compare two numbers from billions to thousandths using the symbols $>$, $=$ or $<$, and justify the solution. • Understand that in a multi-digit number, a digit represents $1/10$ times what it would represent in the place to its left. • Evaluate the value of powers of 10 and understand the relationship to the place value system.
1 st Grade				
<p>Understand place value of two-digit numbers.</p>				
<ul style="list-style-type: none"> • Understand that 10 can be thought of as a bundle of 10 ones – called a “ten”. • Understand two-digit numbers are composed of ten(s) and one(s). • Compare two two-digit numbers using the symbols $>$, $=$ or $<$. • Count by 10s to 120 starting at any number. 				

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Number Sense and Operations in Base Ten Continued

1 st Grade	2 nd Grade	3 rd Grade	4 th Grade	5 th Grade
Use place value understanding to add and subtract.	Use place value understanding and properties of operations to add and subtract.	X	Use place value understanding and properties of operations to perform multi-digit arithmetic with numbers up to one million.	Use place value system understanding to perform operations with multi-digit whole numbers to billions and decimals to thousandths.
<ul style="list-style-type: none"> • Add within 100. • Calculate 10 more or 10 less than a given number mentally without having to count. • Add or subtract a multiple of 10 from another two-digit number, and justify the solution. 	<ul style="list-style-type: none"> • Demonstrate fluency with addition and subtraction within 100. • Add up to four two-digit numbers. • Add or subtract within 1000, and justify the solution. • Use the relationship between addition and subtraction to solve problems. • Add or subtract mentally 10 or 100 to or from a given number within 1000. 	X	<ul style="list-style-type: none"> • Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers, and justify the solution. • Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, and justify the solution. 	<ul style="list-style-type: none"> • Round numbers from billions to thousandths place. • Add and subtract multi-digit whole numbers and decimals to the thousandths place, and justify the solution. • Multiply multi-digit whole numbers and decimals to the hundredths place, and justify the solution. • Divide multi-digit whole numbers and decimals to the hundredths place using up to two-digit divisors and four-digit dividends, and justify the solution.
	<p style="text-align: center;">Represent and solve problems involving addition and subtraction.</p> <ul style="list-style-type: none"> • Write and solve problems involving addition and subtraction within 100. 	X		

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Number Sense and Operations in Fractions

<p>Fractions can be challenging for many students. The standards in this domain are designed to help students see fractions as numbers, or more specifically as parts of a whole, instead of as foreign symbols that are difficult to comprehend. Students explore the relationship of parts to wholes; the notation involved in expressing fractions; the ordering of fractions based on their values; and the use of visual fraction models in addition, in subtraction, and in limited applications of multiplication and division.</p>				
1 st Grade	2 nd Grade	3 rd Grade	4 th Grade	5 th Grade
Reason with shapes and their attributes.	Reason with shapes and their attributes.	Develop understanding of fractions as numbers.	Extend understanding of fraction equivalence and ordering. (Limit denominators to 2, 3, 4, 5, 6, 8, 10, 12 and 100.)	Understand the relationship between fractions and decimals (denominators that are factors of 100).
<ul style="list-style-type: none"> • Compose and decompose two- and three-dimensional shapes to build an understanding of part-whole relationships and the properties of the original and composite shapes • Partition circles and rectangles into two or four equal shares, and describe the shares and the wholes verbally. 	<ul style="list-style-type: none"> • Apply properties of operations to simplify and to factor linear algebraic expressions with rational coefficients. • Partition circles and rectangles into two, three or four equal shares, and describe the shares and the whole. • Demonstrate that equal shares of identical wholes need not have the same shape. 	<ul style="list-style-type: none"> • Understand a unit fraction as the quantity formed by one part when a whole is partitioned into equal parts. • Describe the numerator as representing the number of pieces being considered. • Describe the denominator as the number of pieces that make the whole. • Understand the whole is the interval from 0 to 1. • Understand the whole is partitioned into equal parts. • Understand a fraction represents the endpoint of the length a given number of partitions from 0. 	<ul style="list-style-type: none"> • Explain and/or illustrate why two fractions are equivalent. • Recognize and generate equivalent fractions. • Compare two fractions using the symbols $>$, $=$ or $<$, and justify the solution. 	<ul style="list-style-type: none"> • Understand that parts of a whole can be expressed as fractions and/or decimals. • Convert decimals to fractions and fractions to decimals. • Compare and order fractions and/or decimals to the thousandths place using the symbols $>$, $=$ or $<$, and justify the solution

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Number Sense and Operations in Fractions Continued

3 rd Grade	4 th Grade	5 th Grade
Develop understanding of fractions as numbers.	Extend understanding of operations on whole numbers to fraction operations.	Perform operations and solve problems with fractions and decimals.
<ul style="list-style-type: none"> • Demonstrate that two fractions are equivalent if they are the same size, or the same point on a number line. • Recognize and generate equivalent fractions using visual models, and justify why the fractions are equivalent. • Compare two fractions with the same numerator or denominator using the symbols $>$, $=$ or $<$, and justify the solution. • Explain why fraction comparisons are only valid when the two fractions refer to the same whole. 	<ul style="list-style-type: none"> • Understand addition and subtraction of fractions as joining/composing and separating/decomposing parts referring to the same whole. • Decompose a fraction into a sum of fractions with the same denominator and record each decomposition with an equation and justification. • Solve problems involving adding and subtracting fractions and mixed numbers with like denominators • Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. • Solve problems involving multiplication of a fraction by a whole number. <p data-bbox="537 792 1192 894">Understand decimal notation for fractions, and compare decimal fractions. (Denominators of 10 or 100)</p> <ul style="list-style-type: none"> • Use decimal notation for fractions with denominators of 10 or 100. • Understand that fractions and decimals are equivalent representations of the same quantity. • Read, write and identify decimals to the hundredths place using number names, base ten numerals and expanded form. • Compare two decimals to the hundredths place using the symbols $>$, $=$ or $<$, and justify the solution. 	<ul style="list-style-type: none"> • Estimate results of sums, differences and products with fractions and decimals to the thousandths. • Estimate the size of the product based on the size of the two factors. • Explain why multiplying a given number by a fraction greater than 1 results in a product larger than the given number. • Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number. • Explain why multiplying the numerator and denominator by the same number is equivalent to multiplying the fraction by 1. • Solve problems involving addition and subtraction of fractions and mixed numbers with unlike denominators, and justify the solution. • Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths. • Calculate and interpret the product of a fraction by a whole number and a whole number by a fraction. • Calculate and interpret the product of two fractions less than one. • Calculate and interpret the quotient of a unit fraction by a non-zero whole number. • Calculate and interpret the quotient of a whole number by a unit fraction.

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Relationships and Algebraic Thinking

The Progression in Relationships and Algebraic Thinking deals with the basic operations—the kinds of quantitative relationships they model and consequently the kinds of problems they can be used to solve as well as their mathematical properties and relationships. Although most of the standards organized under the RA heading involve whole numbers, the importance of the Progression is much more general because it describes concepts, properties, and representations that extend to other number systems, to measures, and to algebra.

Kindergarten	1 st Grade	2 nd Grade	3 rd Grade	4 th Grade	5 th Grade
Understand addition as putting together, and subtraction as taking apart.	Represent and solve problems involving addition and subtraction.	Add and subtract within 20.	Represent and solve problems involving multiplication and division.	Use the four operations with whole numbers to solve problems.	Represent and analyze patterns and relationships.
<ul style="list-style-type: none"> • Represent addition and subtraction within 10. • Demonstrate fluency for addition and subtraction within 5. • Decompose numbers less than or equal to 10 in more than one way. • Make 10 for any number from 1 to 9. 	<ul style="list-style-type: none"> • Use addition and subtraction within 20 to solve problems. • Solve problems that call for addition of three whole numbers whose sum is within 20. • Develop the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. • Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. 	<ul style="list-style-type: none"> • Demonstrate fluency with addition and subtraction within 20. 	<ul style="list-style-type: none"> • Interpret products of whole numbers. • Interpret quotients of whole numbers. • Describe in words or drawings a problem that illustrates a multiplication or division situation. • Use multiplication and division within 100 to solve problems. • Determine the unknown number in a multiplication or division equation relating three whole numbers. 	<ul style="list-style-type: none"> • Multiply or divide to solve problems involving a multiplicative comparison. • Solve multi-step whole number problems involving the four operations and variables and using estimation to interpret the reasonableness of the answer. • Solve whole number division problems involving variables in which remainders need to be interpreted, and justify the solution. 	<ul style="list-style-type: none"> • Generate two numeric patterns given two rules. • Translate two numeric patterns into two sets of ordered pairs. • Graph numeric patterns on the Cartesian coordinate plane. • Identify the relationship between two numeric patterns. • Write a rule to describe or explain a given numeric pattern.

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Relationships and Algebraic Thinking Continued

1 st Grade	2 nd Grade	3 rd Grade	4 th Grade	5 th Grade
Understand and apply properties of operations and the relationship between addition and subtraction.	Develop foundations for multiplication and division.	Multiply and divide within 100.	Generate and analyze patterns.	Use the four operations to represent and solve problems.
<ul style="list-style-type: none"> Use properties as strategies to add and subtract. Demonstrate that subtraction can be solved as an unknown-addend problem. 	<ul style="list-style-type: none"> Determine if a set of objects has an odd or even number of members. Count by 2s to 100 starting with any even number. Express even numbers as pairings/groups of 2, and write an expression to represent the number using addends of 2. Express even numbers as being composed of equal groups and write an expression to represent the number with 2 equal addends. Find the total number of objects arranged in a rectangular array with up to 5 rows and 5 columns, and write an equation to represent the total as a sum of equal addends. 	<ul style="list-style-type: none"> Multiply and divide with numbers and results within 100 using strategies such as the relationship between multiplication and division or properties of operations. Know all products of two one-digit numbers. Demonstrate fluency with products within 100. 	<ul style="list-style-type: none"> Generate a number pattern that follows a given rule. Use words or mathematical symbols to express a rule for a given pattern. 	<ul style="list-style-type: none"> Solve and justify multi-step problems involving variables, whole numbers, fractions and decimals.
Add and subtract within 20.		Use the four operations to solve word problems.		
<ul style="list-style-type: none"> Add and subtract within 20. Demonstrate fluency with addition and subtraction within 10 		<ul style="list-style-type: none"> Write and solve two-step problems involving variables using any of the four operations. Interpret the reasonableness of answers using mental computation and estimation strategies including rounding. Identify arithmetic patterns and explain the patterns using properties of operations. 		

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Geometry and Measurement

Geometric and spatial thinking are important in and of themselves, because they connect mathematics with the physical world, and play an important role in modeling phenomena whose origins are not necessarily physical, for example, as networks or graphs. They are also important because they support the development of number and arithmetic concepts and skills. Geometric measurement connects the two most critical domains of early mathematics, geometry and number, with each providing conceptual support to the other. Measurement is the process of assigning a number to a magnitude of some attribute shared by some class of objects, such as length, relative to a unit.

Kindergarten	1 st Grade	2 nd Grade	3 rd Grade	4 th Grade	5 th Grade
Reason with shapes and their attributes.	Reason with shapes and their attributes.	Reason with shapes and their attributes.	Reason with shapes and their attributes.	Classify 2D shapes by properties of their lines and angles.	Classify two- and three-dimensional geometric shapes.
<ul style="list-style-type: none"> Describe several measurable attributes of objects. Compare the measurable attributes of two objects 	<ul style="list-style-type: none"> Distinguish between defining attributes versus non-defining attributes; build and draw shapes that possess defining attributes. Compose and decompose two- and three-dimensional shapes to build an understanding of part-whole relationships and the properties of the original and composite shapes. Recognize two- and three-dimensional shapes from different perspectives and orientations. 	<ul style="list-style-type: none"> Recognize and draw shapes having specified attributes, such as a given number of angles or sides. Identify triangles, quadrilaterals, pentagons, hexagons, circles and cubes. Identify the faces of three-dimensional objects. Partition a rectangle into rows and columns of same-size squares and count to find the total number of squares. 	<ul style="list-style-type: none"> Understand that shapes in different categories may share attributes and that the shared attributes can define a larger category. Distinguish rhombuses and rectangles as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to these subcategories. 	<ul style="list-style-type: none"> Draw and identify points, lines, line segments, rays, angles, perpendicular lines and parallel lines. Classify two-dimensional shapes by their sides and/or angles. Construct lines of symmetry for a two-dimensional figure. 	<ul style="list-style-type: none"> Understand that attributes belonging to a category of figures also belong to all subcategories. Classify figures in a hierarchy based on properties. Analyze and describe the properties of prisms and pyramids.

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Geometry and Measurement Continued

Kindergarten	1 st Grade	2 nd Grade	3 rd Grade	4 th Grade	5 th Grade
Work with time and money.	Measure lengths in non-standard units.	Measure and estimate lengths in standard units.	Solve problems involving the measurement of time, liquid volumes and weights of objects.	Understand the concepts of angle and measure angles.	Understand and compute volume.
<ul style="list-style-type: none"> • Demonstrate an understanding of concepts of time and devices that measure time. • Name the days of the week. • Identify pennies, nickels, dimes and quarters. • 	<ul style="list-style-type: none"> • Order three or more objects by length. • Compare the lengths of two objects indirectly by using a third object. • Demonstrate the ability to measure length or distance using objects. • 	<ul style="list-style-type: none"> • Measure the length of an object by selecting and using appropriate tools. • Analyze the results of measuring the same object with different units. • Estimate lengths using units of inches, feet, yards, centimeters and meters. • Measure to determine how much longer one object is than another. • 	<ul style="list-style-type: none"> • Tell and write time to the nearest minute. • Estimate time intervals in minutes. • Solve problems involving addition and subtraction of minutes. • Measure or estimate length, liquid volume and weight of objects. • Use the four operations to solve problems involving lengths, liquid volumes or weights given in the same units. • 	<ul style="list-style-type: none"> • Identify and estimate angles and their measure. • Draw and measure angles in whole-number degrees using a protractor. • 	<ul style="list-style-type: none"> • Understand the concept of volume and recognize that volume is measured in cubic units. • Describe a cube with edge length 1 unit as a “unit cube” and is said to have “one cubic unit” of volume and can be used to measure volume. • Understand that the volume of a right rectangular prism can be found by stacking multiple layers of the base. • Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for volume of right rectangular prisms with whole-number edge lengths •

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Geometry and Measurement Continued

Kindergarten	1 st Grade	2 nd Grade	3 rd Grade	4 th Grade	5 th Grade
Analyze squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres.	Work with time and money.	Relate addition and subtraction to length.	Understand concepts of area.	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	Graph points on the Cartesian coordinate plane within the first quadrant to solve problems.
<ul style="list-style-type: none"> Identify shapes and describe objects in the environment using names of shapes, recognizing the name stays the same regardless of orientation or size. Describe the relative positions of objects in space. Identify and describe the attribute of shapes, and use the attributes to sort a collection of shapes. Draw or model simple two-dimensional shapes Compose simple shapes to form larger shapes using manipulatives. 	<ul style="list-style-type: none"> Tell and write time in hours and half-hours using analog and digital clocks. Know the value of a penny, nickel, dime and quarter. 	<ul style="list-style-type: none"> Use addition and subtraction within 100 to solve problems involving lengths that are given in the same units. Represent whole numbers as lengths on a number line, and represent whole-number sums and differences within 100 on a number line. 	<ul style="list-style-type: none"> Calculate area by using unit squares to cover a plane figure with no gaps or overlaps. Label area measurements with squared units. Demonstrate that tiling a rectangle to find the area and multiplying the side lengths result in the same value. Multiply whole-number side lengths to solve problems involving the area of rectangles. Find rectangular arrangements that can be formed for a given area. Decompose a rectangle into smaller rectangles to find the area of the original rectangle. 	<ul style="list-style-type: none"> Know relative sizes of measurement units within one system of units. Convert measurements in a larger unit in terms of a smaller unit. Use the four operations to solve problems involving distances, intervals of time, liquid volume, weight of objects and money. Apply the area and perimeter formulas for rectangles to solve problems. 	<ul style="list-style-type: none"> Define a first quadrant Cartesian coordinate system. Represent the axes as scaled perpendicular number lines that both intersect at 0, the origin. Identify any point on the Cartesian coordinate plane by its ordered pair coordinates. Define the first number in an ordered pair as the horizontal distance from the origin. Define the second number in an ordered pair as the vertical distance from the origin. Plot and interpret points in the first quadrant of the Cartesian coordinate plane.

HSD Elementary Mathematics Learning Progressions by Domain

Geometry and Measurement Continued

2 nd Grade	3 rd Grade	5 th Grade
Work with time and money.	Understand concepts of perimeter.	Solve problems involving measurement and conversions within a measurement system.
<ul style="list-style-type: none"> • Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. • Describe a time shown on a digital clock as representing hours and minutes, and relate a time shown on a digital clock to the same time on an analog clock. • Find the value of combinations of dollar bills, quarters, dimes, nickels and pennies, using \$ and ¢ appropriately. • Find combinations of coins that equal a given amount. 	<ul style="list-style-type: none"> • Solve problems involving perimeters of polygons. • Understand that rectangles can have equal perimeters but different areas, or rectangles can have equal areas but different perimeters. • 	<ul style="list-style-type: none"> • Convert measurements of capacity, length and weight within a given measurement system. • Solve multi-step problems that require measurement conversions. •

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Data and Statistics

Kindergarten work with data uses counting and order relations. First- and second-graders solve addition and subtraction problems in a data context. In Grades 3–5, work with data is closely related to the number line, fraction concepts, fraction arithmetic, and solving problems that involve the four operations.

Kindergarten	1 st Grade	2 nd Grade	3 rd Grade	4 th Grade	5 th Grade
Classify objects and count the number of objects in each category.	Represent and interpret data.	Represent and interpret data.	Represent and analyze data.	Represent and analyze data.	Represent and analyze data.
<ul style="list-style-type: none"> Classify objects into given categories; count the number of objects in each category. Compare category counts using appropriate language. 	<ul style="list-style-type: none"> Collect, organize and represent data with up to three categories. Draw conclusions from object graphs, picture graphs, T-charts and tallies. 	<ul style="list-style-type: none"> Create a line plot to represent a set of numeric data, given a horizontal scale marked in whole numbers. Generate measurement data to the nearest whole unit, and display the data in a line plot. Draw a picture graph or a bar graph to represent a data set with up to four categories. Solve problems using information presented in line plots, picture graphs and bar graphs. Draw conclusions from line plots, picture graphs and bar graphs. 	<ul style="list-style-type: none"> Create frequency tables, scaled picture graphs and bar graphs to represent a data set with several categories. Solve one- and two-step problems using information presented in bar and/or picture graphs Create a line plot to represent data. Use data shown in a line plot to answer questions. 	<ul style="list-style-type: none"> Create a frequency table and/or line plot to display measurement data. Solve problems involving addition and subtraction by using information presented in a data display. Analyze the data in a frequency table, line plot, bar graph or picture graph. 	<ul style="list-style-type: none"> Create a line graph to represent a data set, and analyze the data to answer questions and solve problems. Create a line plot to represent a given or generated data set, and analyze the data to answer questions and solve problems, recognizing the outliers and generating the median.