Performance Tasks for Part 4

Loci and Conics

THE FOLLOWING ACTIVITIES SHOULD EACH TAKE LESS THAN A PERIOD TO COMPLETE.

1. Orbiting the Earth

Satellites normally orbit the Earth in an elliptical orbit, with the centre of the Earth being one focal point of the orbit. The point where the satellite is nearest the Earth is the **perigee** and the point at which it is farthest from the Earth is the **apogee**. Suppose a satellite is 620 km from the Earth's surface at its perigee and 1200 km from the Earth's surface at its apogee. Determine the equation for the orbit of the satellite. Include a diagram with your solution. (Note that the Earth has a radius of about 6.34×10^3 km.)

2. Conics and Foci

The ratio of the distance between the centre of a conic and one of its foci to the distance between the centre and one of the vertices is called the **eccentricity** of a conic.

(a) Create several equations of ellipses and several equations of hyperbolas and determine the eccentricity of each ellipse and hyperbola.

(b) Make a conjecture comparing the eccentricity of ellipses to the eccentricity of hyperbolas.

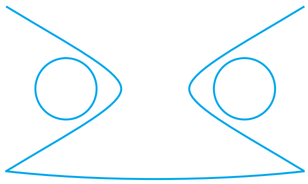
(c) State the equations of one other hyperbola and one other ellipse and determine their eccentricities. Does your conjecture prove true?

(d) Investigate the eccentricity of the other conics and create a summary of your findings.

(e) Submit a report showing all of your work that supports your summary.

3. What Are the Equations?

(a) Create equations of conics and lines so that the given diagram will result when graphed:



(b) State the equations and draw the graphs of your equations.

(c) Describe the process you went through to determine the equations. Include key words such as foci, vertices, radius, and centre.

(d) Add another feature to this diagram by using the equation of a line or a conic. This new line or conic must intersect one of the given graphs. Explain the relationship of this new equation and graphs to the others.

(e) Find at least one intersection point in your graph.

4. What Are the Solutions?

Create both graphical and algebraic solutions to these problems:

(a) Find an equation of the circle that passes through points (10, 9), (4, -5), and (0, 5).

(b) Find the distances from the foci of the ellipse $16x^2 + 25y^2 = 400$ to a point on the curve whose x-coordinate is 2.

(c) Find the equation of two parabolas in standard position that pass through $P(4, -2)$.

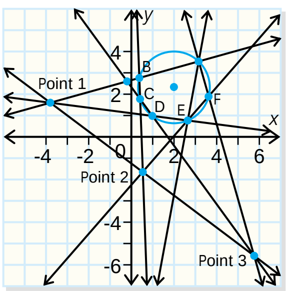
(d) Show that the hyperbola $9x^2 - 16y^2 = 144$ and the ellipse $3x^2 + 4y^2 = 300$ have the same focus.

THE FOLLOWING ACTIVITIES COULD EACH TAKE MORE THAN A PERIOD TO COMPLETE.

5. Blaise Pascal

Blaise Pascal (1623–1662) was a great French mathematician and philosopher. The computer language Pascal and the metric unit the pascal are both named for him. One of Pascal's theorems deals with circles, ellipses, hyperbolas, and parabolas. To investigate the theorem, try the following. You may find it useful to use *The Geometer's Sketchpad* for your exploration.

(a) Pick any six points on a conic and label them A through F, in order of position.



(b) Draw lines that connect consecutive points. In other words, draw lines *AB*, *BC*, *CD*, *DE*, *EF*, and *FA*.

(c) Mark the intersection of these sets of lines.

i. Mark the intersection of *AB* and *DE* as point 1.

ii. Mark the intersection of *BC* and *EF* as point 2.

iii. Mark the intersection of *CD* and *FA* as point 3.

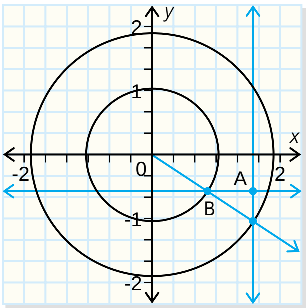
(d) What do you notice about these three points?

(e) Test your conjecture with the other conics.

(f) Submit a report with the graphs and equations of several conics that you investigated. Summarise and justify your results.

6. Constructing an Ellipse

There are many different ways to construct an ellipse. One way is to use two concentric circles with two perpendicular lines drawn through the centre as in the diagram.



When *B* is dragged around the circle, point *A* traces an ellipse. Experiment with this construction and then prove that this construction produces an ellipse.