Title: Pack It Up!

Brief Overview:

Pack It Up! consists of three lessons designed to address aspects of the NCTM geometry standard for grades 6-8. In the first lesson, the students will analyze different solids and investigate their nets. The second lesson is designed to have students discover how to find the surface area and volume of rectangular prisms. In the third lesson, the students will apply their knowledge of these concepts to solve a real world problem involving package design. These particular lessons would be the first in a series of lessons on geometric solids.

NCTM Content Standard/National Science Education Standard:

NCTM Content Standards for Grades 6-8: Geometry and Measurement

- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume
- Develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders

Grade/Level:

6-8

Duration/Length:

Three 86-minute periods

Student Outcomes:

The students will:

Lesson 1: Draw nets for prisms, pyramids, cylinders, and cones

Lesson 2: Develop and apply the formula to derive the surface area and volume of a rectangular prism

Lesson 3: Develop the concept of minimizing the surface area of a rectangular prism with a given volume

Materials and Resources:

Lesson 1

- Pictures of real world three-dimensional shapes (examples can include, but are not limited to a house, tent, can, etc.)
- Cereal box for teacher demonstration (mini-cereal boxes would be ideal for the students to use during the demonstration)
- Plastic solids with nets included (or an overhead of each net if they are not included in the solid)
- Nets worksheet (worksheet 1)
- Activity worksheet: Faces, Vertices, and Edges (worksheet 2)
- Copy of nets of cardstock
- Tape
- Scissors

Lesson 2

- Anticipation guide: Surface Area and Volume of Rectangular Prisms (worksheet 3)
- Cereal box from previous day
- Group-Pair-Solo Problems (worksheet 4)
- Classwork : Surface Area and Volume of Rectangular Prisms (worksheet 5)
- Journal Entry (worksheet 6)

Lesson 3

- Placemat template
- A set of 36 cubes for each group of four students
- Project worksheet: Pack It Up! (worksheet 7)
- Poster paper
- Markers
- Assessment: Movie Theater Popcorn (worksheet 8)
- Rulers
- Cardboard or paper
- Quiz (worksheet 9)

Development/Procedures:

Lesson 1: Working with Nets

Preassessment and Launch

Students will identify familiar shapes in real world pictures. For example, if a picture of a house is shown, students should identify squares (windows), rectangles (door), triangles (roof), and circles (doorknobs). Characteristics of each shape will be discussed.

Teacher Facilitation

The teacher will begin the lesson by showing the students an actual cereal box. If possible, the students should get a mini-cereal box to use during the demonstration. The teacher could provide the mini-cereal boxes or the students could be asked to bring one in. If this is not possible, the students can just watch the teacher. The teacher will ask prompting questions about the parts that make up the box, such as the top, bottom, sides, front, and back. The teacher will introduce the correct vocabulary of **faces**, **vertices**, and **edges**. The teacher will proceed to cut open the box, flatten it out, and check for any prior student knowledge of **nets**. After some student discussion, the teacher will define a **net** and students will copy the vocabulary.

Student Application

The class will break into groups of four and obtain a copy of the nets worksheet (worksheet 1). Each group will receive a plastic solid that has a net of the solid inside of it. Most of these sets will have a triangular prism, cube, rectangular prism, hexagonal prism, triangular pyramid, square or rectangular pyramid, cylinder, and a cone. They will have two to three minutes to draw the solid, their prediction of what the net looks like, and then what the actual net looks like. The students should not open the plastic solids and take out the net until they have drawn their prediction of the net. If these particular plastic solids are not available, the teacher could show an overhead of the correct nets after the students have drawn all of their predictions. The groups should rotate the solids until each group has seen all of the solids. A Teach Timer is a great tool to keep students on task during this activity. Once the activity is completed, the teacher could have each group present one of the solids to the class.

Embedded Assessment

The teacher can assess student understanding of nets by walking around the room and observing the drawings while the students are working in groups. The teacher could also assess understanding as the students present their findings on the nets of the solids.

Reteaching/Extension

If students are still having difficulty with nets, they could do an activity where they work backwards from the previous activity. The teacher could show the students the net of a solid, discuss it and have them identify the solid, and then have the students cut out the net and construct the solid. If all of the students understand nets, they can complete the faces, edges, and vertices activity (worksheet 2). This worksheet has them identify the number of faces, vertices, and edges of different prisms and pyramids. By filling in the chart, it is hoped that they would discover the relationship between the faces, vertices, and edges. They should also see the pattern that can be used to find the number of faces, vertices, and edges for any prism or pyramid. If time permits, these activities could be used as part of the actual lesson.

Lesson 2: Surface Area and Volume of Rectangular Prisms

Preassessment and Launch

Students will complete an anticipation guide on surface area and volume (worksheet 3). The purpose of this guide is to assess their prior knowledge of these concepts.

Teacher Facilitation

The teacher will begin by reviewing what was learned yesterday about solids, in particular what was learned about rectangular prisms. The purpose of this lesson is to have the students derive the formulas for surface area and volume of rectangular prisms. The teacher will begin the discussion by using the net of the cereal box from yesterday. The discussion will proceed with the teacher asking leading questions about how to find the area of each face and asking the students what it is called when we find the area of all the faces together **(surface area)**. The teacher will continue by asking students how the surface area would be calculated. It is expected that students will say that the area of each face has to be found and then added together. The teacher could already have the measures of the sides of the box labeled or a student could come up and measure the sides. Then the students will find the area of each part of the box. After making these calculations, the students should recognize that the sides have equal areas, the top and bottom have equal areas, and the front and back have equal areas. This will lead to the derivation of the formula $SA = 2lw + 2wh + 2lh$ or $2\,[lw + wh + lh]$, where SA is the surface area, l is the length, w is the width, and h is the height of the prism. The students will calculate the final answer for the surface area of the cereal box. The teacher will start a discussion about what surface area tells us by prompting the students with the question, "Why would we need to know the surface area of a box?" Students should come up with answers such as wrapping gifts, constructing packages, icing cakes, and painting a fence. Once surface area has been covered and the students feel comfortable, the teacher will start a discussion of volume. The teacher will present the question, "What is the term that describes filling the box?" The students should come up with the term **volume**. Instead of just giving them the formula, the teacher will again try to have them discover it. Through the use of leading questions by the teacher, the students will discover that you must first cover the bottom of the box, which is the area of the base, and you continue covering that until the box is full. Each time they cover a layer they are adding another base area. The number of total base areas that is needed to fill the box is equal to the height. Therefore, taking the area of the base and multiplying it by the height will find the volume of a rectangular prism. Since the base is a rectangle, the area is found by multiplying the length and the width. Therefore, the formula $V = lwh$, where V is the volume, l is the length, w is the width, and h is the height is derived.

Student Application

Students will complete a Group-Pair-Solo activity. In the group-pair-solo activity, the students are first given one problem to work on for two to three minutes in a group of four. When all groups have finished, there is a whole class discussion of the problem. The teacher then gives the students a second problem to be solved in pairs. When all pairs have finished, there is a whole class discussion of the problem. Finally, the teacher presents a third problem that the students must solve individually. Worksheet 4 shows problems that can be used, although the problems would not be given to the students on a worksheet. The teacher can write the problems on the board one at a time for the students to solve or have an overhead prepared and show one problem at a time. When all students are finished, there can be a class discussion of the problems or the teacher may want to collect the problems.

Embedded Assessment

The students will complete the worksheet entitled Surface Area and Volume of Rectangular Prism (worksheet 4). The teacher can assess student understanding as they walk around while students are working. This worksheet could also be collected and graded. The teacher will also have the students refer back to the anticipation guide to see if they still agree with their original answers.

Reteaching/Extension

The students will complete a journal question (worksheet 5). A worksheet of this question is included, but the teacher may just decide to write it on the chalkboard or overhead. The topic of the journal entry is to explain to an absent friend everything that was learned that day about surface area and volume of rectangular prisms. If any students are still having difficulty with surface area and volume, the teacher can provide extra assistance at this time.

Lesson 3: Designing Packages

Preassessment and Launch

The students will complete a placemat activity in groups of four. The students will first individually brainstorm careers that involve surface area and volume. Each student records his/her ideas in his/her area of the placemat. Once students are finished brainstorming individually, the group of students share their ideas with each other. Finally, they record all of the careers that they came up with in the center of the placemat. The teacher can then have each group share their ideas.

Teacher Facilitation

The teacher will first review how to determine the surface area and volume of a rectangular prism. The teacher will then introduce the project that the students will be working on today. In this project, the students will work together in

groups of four as design engineers for the Smith and Jones Company, which produces and distributes Soft Plus brand tissues. The design engineer is in charge of designing the most cost effective box in which to ship the boxes of tissues. The tissue boxes are cubes and they want to ship 36 of them in one larger box. In order to simulate this scenario, the students will be given 36 cubes, preferably cubes that interlock, and they will investigate different shaped boxes that could hold all of the cubes. The students will record their information on the project worksheet (worksheet 6). When all of the groups are finished, the teacher will start a discussion of the results.

Student Application

The students will begin building boxes that can be made with the 36 cubes. They will record all of their data onto the project worksheet, including the dimensions, surface area, and volume of the box. The students must investigate all of the different possibilities of boxes that can be made with the 36 cubes. Once the groups have had sufficient time to complete that task, one person from each group will transfer the results to a poster board to be displayed in the room. Each group will present their results to the class. A discussion will then be held about the similarities and/or differences between the results of each group. The students should see that the volume is the same for each box since there are 36 cubes that have to fit in the box. They should also see that for a fixed volume, the surface area is minimized when the box is a cube, or in this case, when it is as close to a cube as possible. As the design engineers, they have to be concerned with the minimum surface area if they want to minimize the cost to the company of producing the boxes in which the tissues will be shipped.

Embedded Assessment

The students will complete the assessment worksheet entitled Movie Theater Popcorn (worksheet 7). The teacher can assess student understanding of the lesson by walking around as they complete this worksheet or the worksheet can be collected and graded. The teacher could also have students complete it individually first and then compare their answers with classmates to ensure that they arrived at the correct answers.

Reteaching/Extension

In order to reteach, extend, and further assess their understanding of the lesson, the students will be presented with the following scenario: The company has changed their minds as to how many boxes of tissues they want to ship together in a box. They now only want to ship 24 tissue boxes together in one larger box. The students will design and create the package that is the most cost effective for the company. The students will actually construct the shipping box out of cardboard or paper. If time allows, they could decorate the box and have fun with it!

Summative Assessment:

The students will complete a quiz that covers what was learned in the three lessons (worksheet 8). The material includes identifying the number of faces, vertices, and edges of different solids, drawing the nets of a variety of solids, and determining the surface area and volume of rectangular prisms. The quiz includes selected response (SR), student produced response (SPR), brief constructed response, (BCR) and extended constructed response (ECR) questions. After the entire unit is completed and students have learned how to determine surface area and volume of pyramids, cylinders, and cones, a culminating project could be given that relates to the original scenario of lesson three. The students could work in groups to create their own product and then create the packaging for that product. By knowing about the different solids, they may or may not choose to use a rectangular prism for their package. It would be a fun and interesting way for the students to really get involved in the mathematics behind product and package design!

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Nets Worksheet **Name _____**

Name and Sketch of Solid	Your Prediction of the Net	Actual Net

Activity: Faces, Vertices, and Edges

Name _____
Class _____
Date _____

Type of Solid	Number of Faces	Number of Vertices	Number of Edges
Triangular Prism			
Rectangular Prism			
Pentagonal Prism			
Hexagonal Prism			
Octagonal Prism			
n-gonal Prism			
Triangular Pyramid			
Rectangular Pyramid			
Pentagonal Pyramid			
Hexagonal Pyramid			
n-gonal Pyramid			

1. Use the pattern to determine the number of faces, vertices, and edges of a 16-gonal prism.

   Faces = _____

   Vertices = _____

   Edges = _____

2. Do you notice a relationship between the number of faces, vertices, and edges of the solids?

ANSWER KEY

Activity: Faces, Vertices, and Edges

Name _____
Class _____
Date _____

Type of Solid	Number of Sides of Base	Number of Faces	Number of Vertices	Number of Edges
Triangular Prism	3	5	6	9
Rectangular Prism	4	6	8	12
Pentagonal Prism	5	7	10	15
Hexagonal Prism	6	8	12	18
Octagonal Prism	8	10	16	24
n-gonal Prism	n	n + 2	2n	3n
Triangular Pyramid	3	4	4	6
Rectangular Pyramid	4	5	5	8
Pentagonal Pyramid	5	6	6	10
Hexagonal Pyramid	6	7	7	12
n-gonal Pyramid	n	n + 1	n + 1	2n

1. Use the pattern from the table to determine the number of faces, vertices, and edges of a 16-gonal prism.

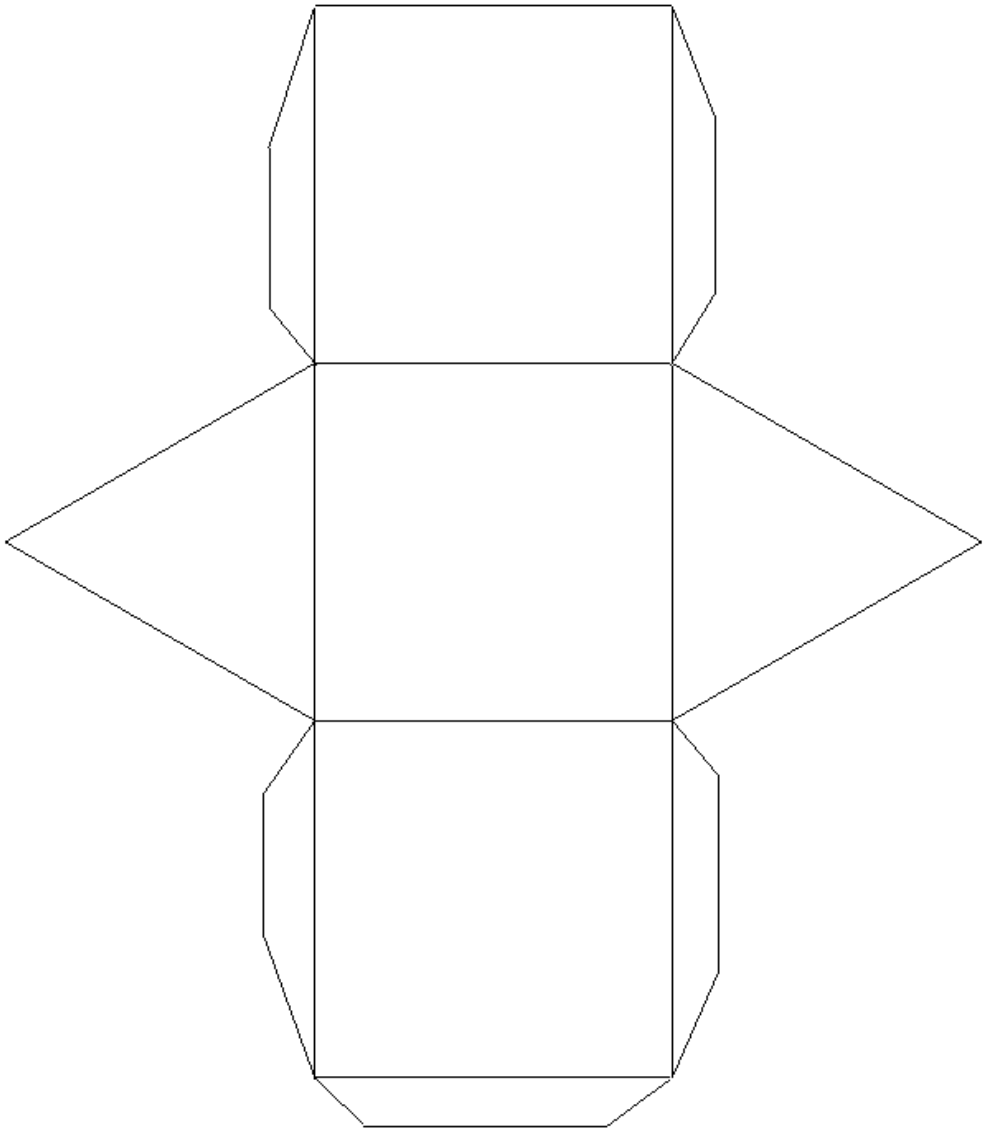
   Faces = _____18_____

   Vertices = _____32_____

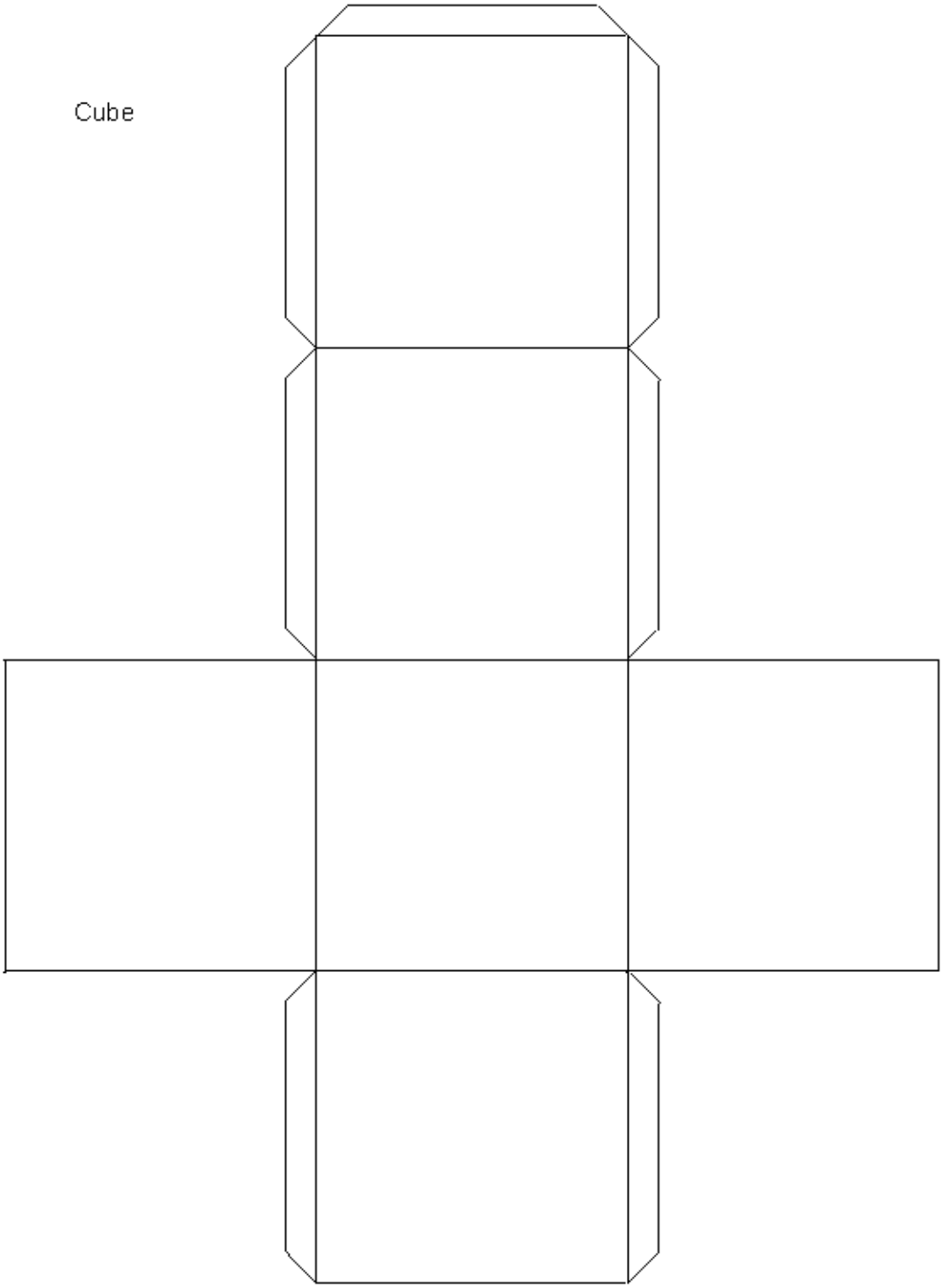
   Edges = _____48_____

2. Do you notice a relationship between the number of faces, vertices, and edges of the solids? F + V - 2 = E

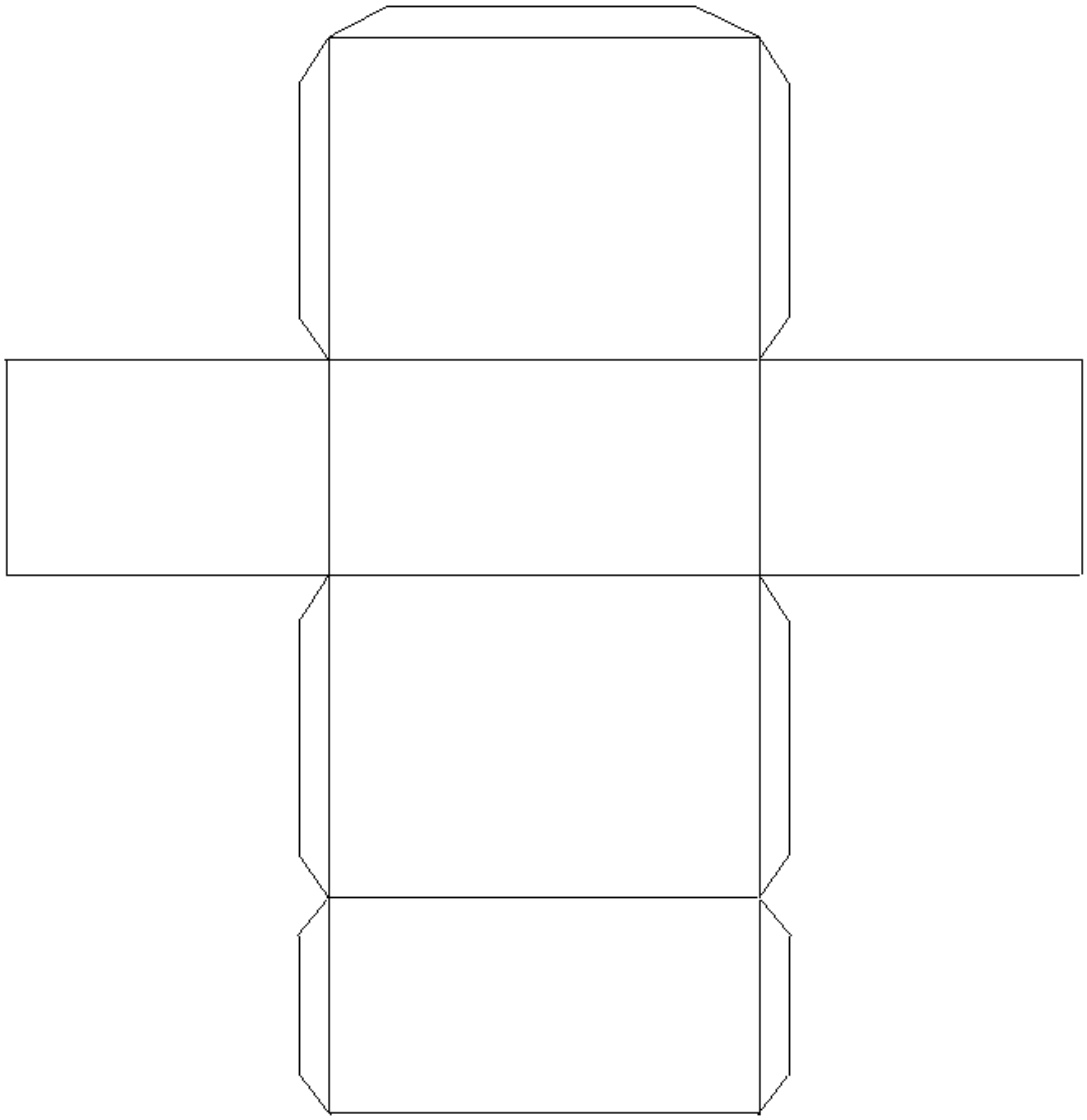
Triangular prism



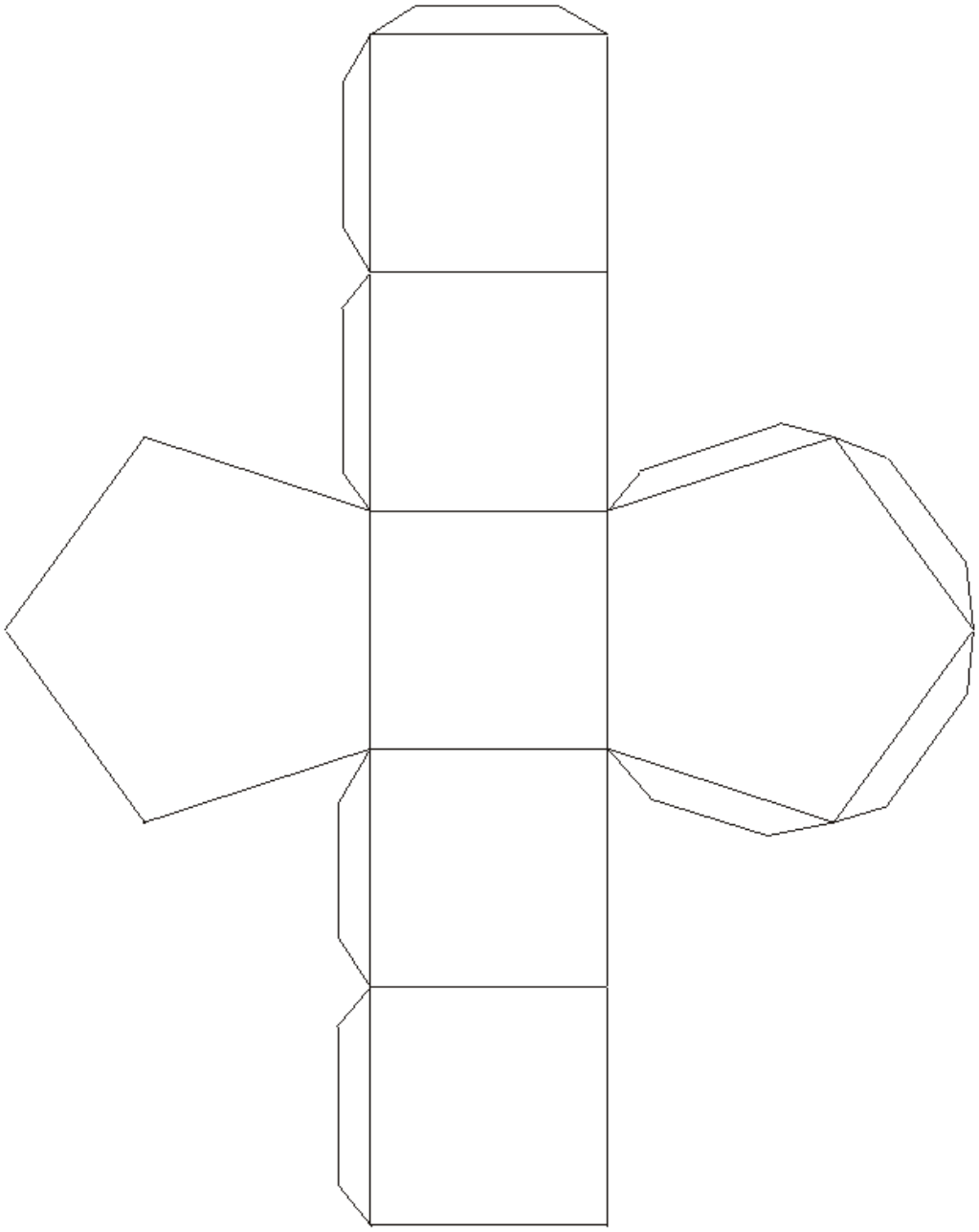
Cube



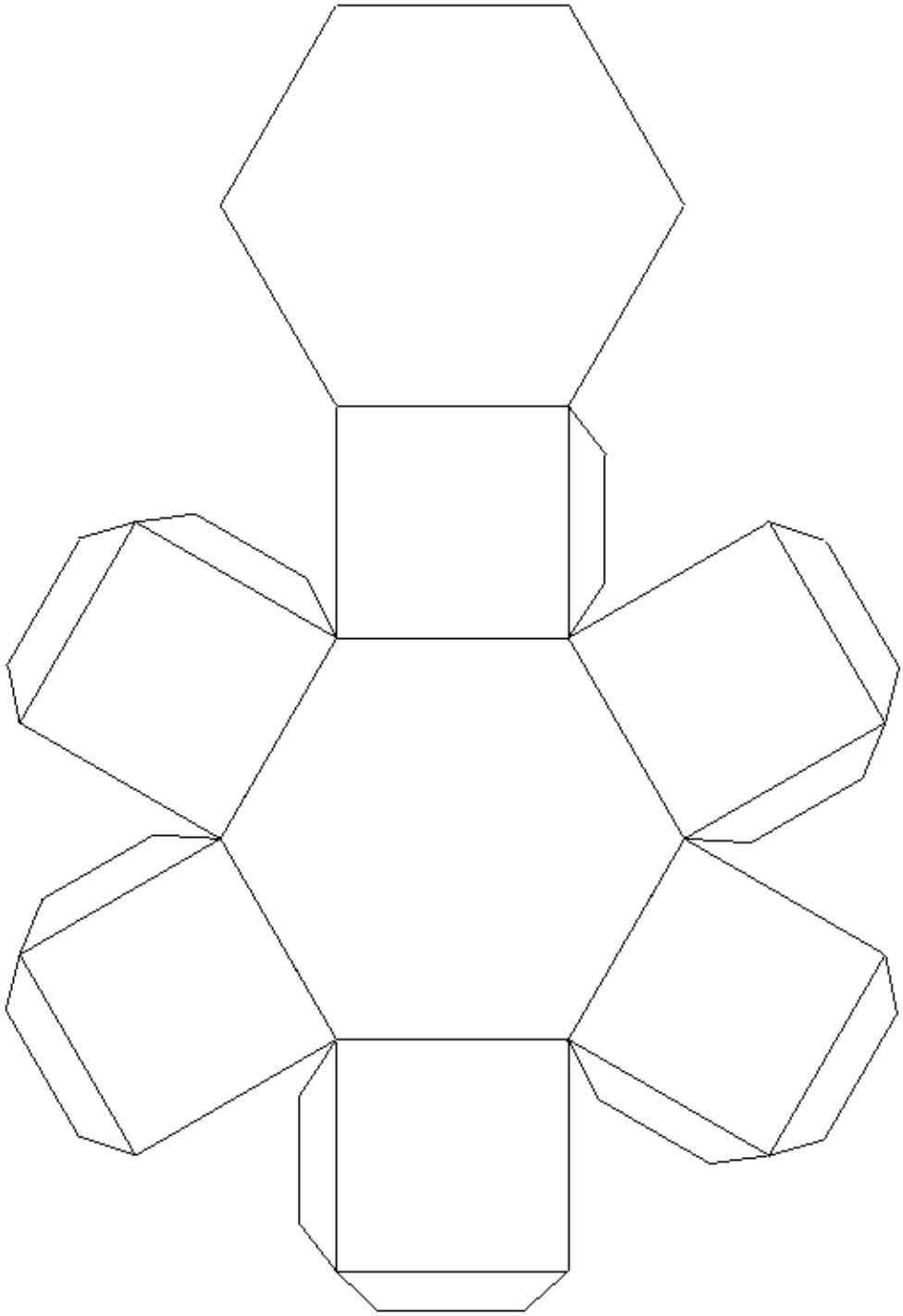
Rectangular Prism



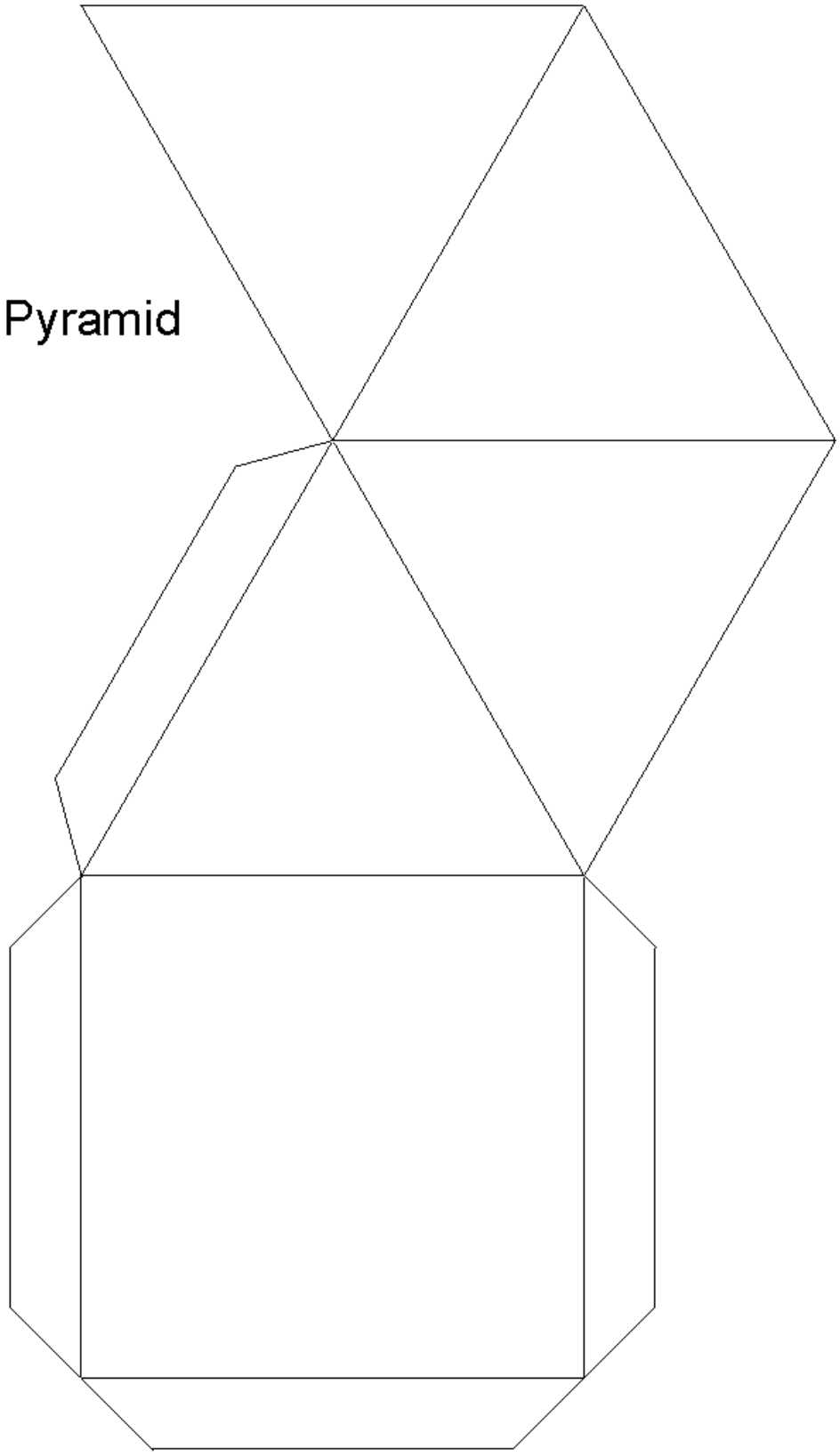
Pentagonal Prism



Hexagonal Prism



Pyramid



Anticipation Guide: Surface Area and Volume of Rectangular Prisms

Name _____
Class _____
Date _____

Directions: For each statement, circle whether you agree or disagree.

Before	***Question***	***After***
Agree or Disagree	1. To find surface area of a rectangular prism you multiply the length, width, and height.	Agree or Disagree
Agree or Disagree	2. Volume is related to "filling" an object.	Agree or Disagree
Agree or Disagree	2. To find the volume of rectangular prism you multiply the length, width, and height	Agree or Disagree
Agree or Disagree	3. If you double the length, width, or height of a rectangular prism, you will double the surface area.	Agree or Disagree
Agree or Disagree	4. The units for volume are square units.	Agree or Disagree

KEY

Anticipation Guide: Surface Area and Volume of Rectangular Prisms

Name _____
Class _____
Date _____

Directions: For each statement, circle whether you agree or disagree.

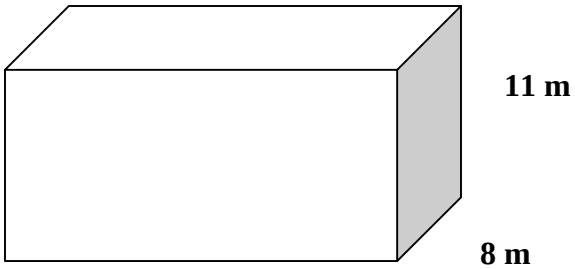
Before	***Question***	***After***
Agree or Disagree	5. To find surface area of a rectangular prism you multiply the length, width, and height.	Agree or Disagree
Agree or Disagree	2. Volume is related to "filling" an object.	Agree or Disagree
Agree or Disagree	6. To find the volume of rectangular prism you multiply the length, width, and height	Agree or Disagree
Agree or Disagree	7. If you double the length, width, or height of a rectangular prism, you will double the surface area.	Agree or Disagree
Agree or Disagree	8. The units for volume are square units.	Agree or Disagree

Various answers

Group-Pair-Solo Problems

Group Problem

Find the volume and surface area of the rectangular prism.



Pair Problem

Michael wants to buy his teacher a present. He has bought a gift box with dimensions 11 in. by 5 in. by 4 in.

Part A: How much wrapping paper does he need to cover the present?

_____

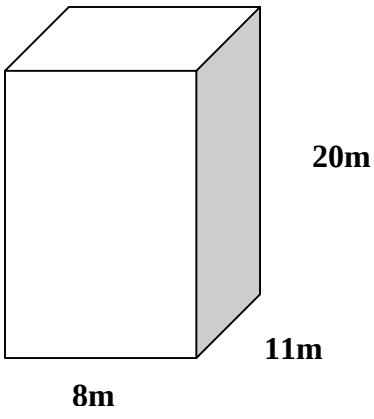
Part B:

- Use what you know about surface area of a rectangular prism to explain how you got your answer. Use words, numbers, or symbols in your answer.

- If he wants to fill the box with erasers for the teacher to use as student incentives, how much would it hold?

Solo Problem

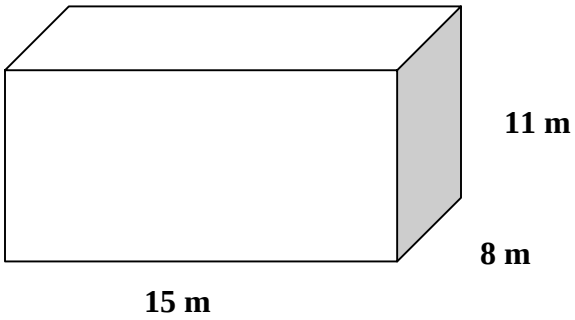
Find the volume and surface area of the rectangular prism.



ANSWER KEY Group-Pair-Solo Problems

Group Problem

Find the volume and surface area of the rectangular prism.



SA= 2 [lw + lh + wh]
SA= 2 [15 • 8 + 15 • 11 + 8 • 11]
SA= 2 [120 + 165 + 88]
SA= 2(373)
SA= 476 m^2

V= lwh
V= 15 • 8 • 11
V= 1320 m^3

Pair Problem

Michael wants to buy his teacher a present. He has bought a gift box with dimensions 11 in. by 5 in. by 4 in.

SA= 2 [lw + lh + wh]
SA= 2 [11•5 + 11•4 + 5•4]
SA= 2 [55+44+20]
SA= 2 [119]
SA= 238 $in.^2$

Part A: How much wrapping paper does he need to cover the present? ___238 in^2_____

Part B:

- Use what you know about surface area of a rectangular prism to explain how you got your answer. Use words, numbers, or symbols in your answer.

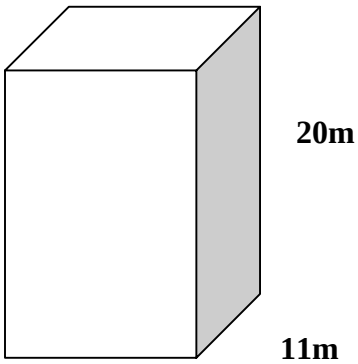
Surface area is the amount of area it takes to cover the surface. I used the formula *SA= 2 [lw + lh + wh]*. I substituted the numbers in for the variables in the formula.
SA= 2 [11• 5 + 11 • 4 + 5 • 4]
SA= 2 [55+44+20], SA= 2 [119], SA= 238 $in.^2$

- If he wants to fill the box with erasers for the teacher to use as student incentives, how much would it hold?

Volume is the amount of space it takes to fill an object I used the formula *V= lwh*.
I substituted the numbers in for the variables in the formula, V= 15 • 8 • 11, V= 1320 m^3.

Solo Problem

Find the volume and surface area of the rectangular prism.



SA= 2 [lw + lh + wh]
SA= 2 [11•8 + 11•20 + 8•20]
SA= 2 [88 + 220 + 160]
SA= 2(468)
SA= 936 m^2

V= lwh
V= 11•8•20
V= 1320 m^3

Classwork: Surface Area and Volume of Rectangular Prisms Name_____
Date _____

A. Determine whether you would need to determine the surface area or volume for each situation.

1) The amount of paint needed to paint a house

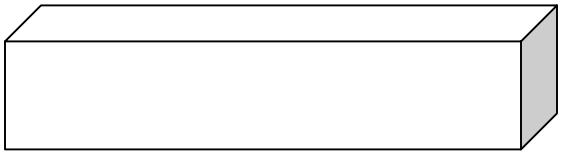
2) The capacity of a swimming pool

3) The amount of ice cream in a cone

4) The amount of wrapping paper needed to wrap a gift

B. Find the surface area and volume of each rectangular prism. Show all of your work!

5)

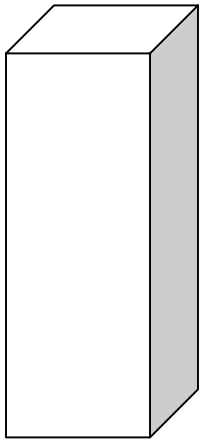


3 in.

3 in.

6 in.

6)

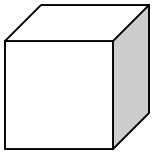


9.5 cm

2.5 cm

4 cm

7)

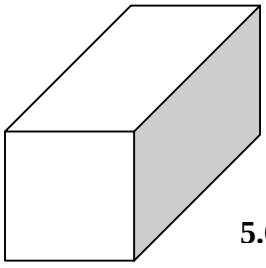


5.2 cm

5.2 cm

5.2 cm

8)



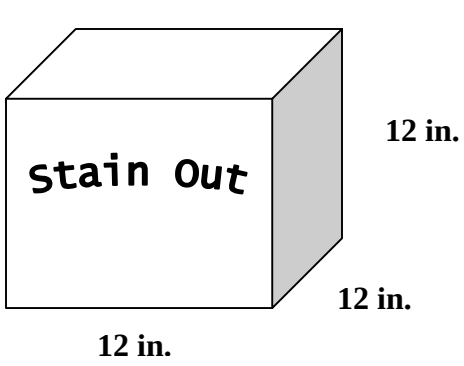
3.2 in.

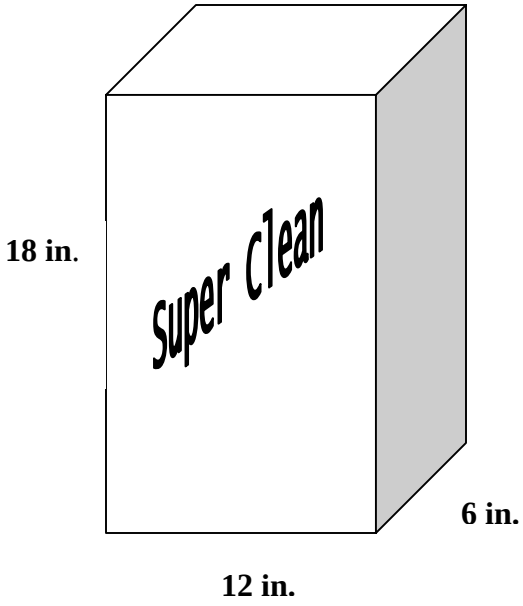
5.6 in.

2.8 in.

C. Read each question carefully and then answer the question. Show all of your work!

9) A store carries two different types of detergent. They are packaged in two different rectangular boxes. These boxes are shown below:





a) Which box has the smallest surface area? Why might the company want to know this?

b) Which box holds more detergent? Why might we want to know this?

c) If Stain Out costs $13.50 per box and Super Clean costs $8.99 per box, which box is the better deal for the consumer?

ANSWER KEY

Classwork: Surface Area and Volume of Rectangular Prisms Name_____

Date _____ Worksheet 5

A. Determine whether you would need to determine the surface area or volume for each situation.

1) The amount of paint needed to paint a house
Surface Area

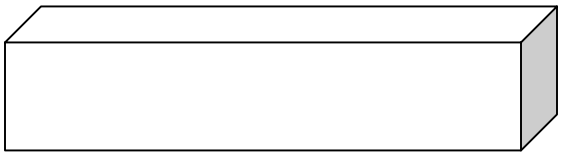
2) The capacity of a swimming pool
Volume

3) The amount of ice cream in a cone
Volume

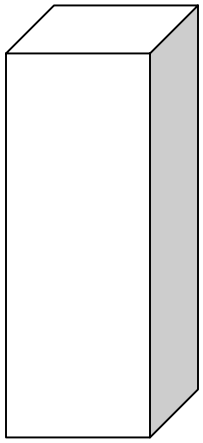
4) The amount of wrapping paper needed to wrap a gift
Surface Area

B. Find the surface area and volume of each rectangular prism. Show all of your work!

5)



6)



5)

SA = 2[lw + wh + lh]
SA = 2[6•3 + 3•3 + 6•3]
SA = 2[18 + 9 + 18]
SA = 2(45)
SA = 90 in.^{2}

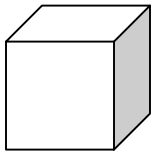
V = lwh
V = 6•3•3
V = 54 in.^{3}

6)

SA = 2[lw + wh + lh]
SA = 2[4•2.5 + 2.5•9.5 + 4•9.5]
SA = 2(71.75)
SA = 143.5 cm^2

V = lwh
V = 4•2.5•9.5
V = 95 cm^3

7)

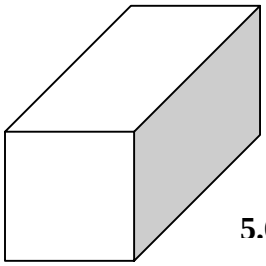


5.2 cm

5.2 cm

5.2 cm

8)



3.2 in.

5.6 in.

2.8 in.

7)

SA = 2[lw + wh + lh]
SA = 2[5.2•5.2 + 5.2•5.2 + 5.2•5.2]
SA = 2[27.04 + 27.04 + 27.04]
SA = 2(81.12)
SA = 162.24 cm^2

V = lwh
V = 5.2•5.2•5.2
V = 140.608 cm^3

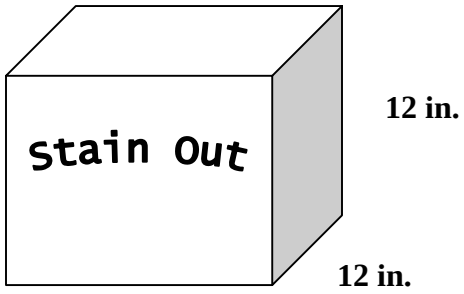
8)

SA = 2[lw + wh + lh]
SA = 2[2.8•5.6 + 5.6•3.2 + 2.8•3.2]
SA = 2[15.68 + 17.92 + 8.96]
SA = 2(42.56)
SA = 85.12 in^2

V = lwh
V = 2.8•5.6•3.2
V = 50.176 in^3

C. Read each question carefully and then answer the question. Show all of your work!

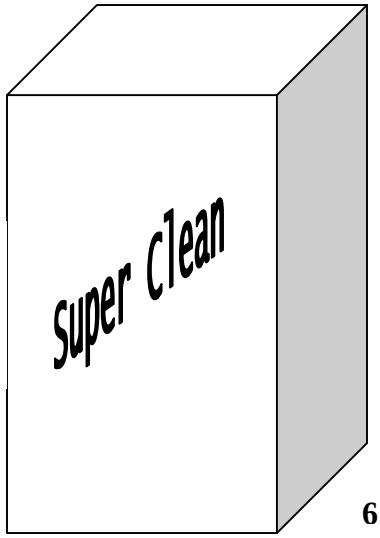
9) A store carries two different types of detergent. They are packaged in two different rectangular boxes. These boxes are shown below:



12 in.

12 in.

12 in.



18 in.

6 in.

12 in.

a) Which box has the smallest surface area? Why might the company want to know this?

Stain Out:
$SA = 2[lw + wh + lh]$
$SA = 2[12 \bullet 12 + 12 \bullet 12 + 12 \bullet 12]$
$SA = 2[144 + 144 + 144]$
$SA = 2(432)$
$SA = 864 \text{ in.}^2$

Super Clean:
$SA = 2[lw + wh + lh]$
$SA = 2[12 \bullet 6 + 6 \bullet 18 + 12 \bullet 18]$
$SA = 2[72 + 108 + 216]$
$SA = 2(396)$
$SA = 792 \text{ in.}^2$

The company would want to know about surface area because that would affect the cost to produce the box.

b) Which box holds more detergent? Why might we want to know this?

Stain Out:
$V = lwh$
$V = 12 \bullet 12 \bullet 12$
$V = 1728 \text{ in}^3$

Super Clean:
$V = lwh$
$V = 12 \bullet 6 \bullet 18$
$V = 1296 \text{ in}^3$

c) If Stain Out costs $13.50 per box and Super Clean costs $8.99 per box, which box is the better deal for the consumer?

Stain Out:

$\$13.50 / 1728 \text{ in}^3 \approx \$.008 \text{ per } 1 \text{ in}^3$

Super Clean:

$\$8.99 / 1296 \text{ in}^3 \approx \$.007 \text{ per in.}^3$

Super Clean is a better deal since it has a smaller unit rate. It costs less per cubic inch than Stain Out does.

Journal Entry

Name _____
Class _____
Date _____

Directions: Your best friend was absent from school today. The teacher has asked you to explain surface area and volume to your friend. Be sure to include all pertinent information, including definitions, formulas, and examples.

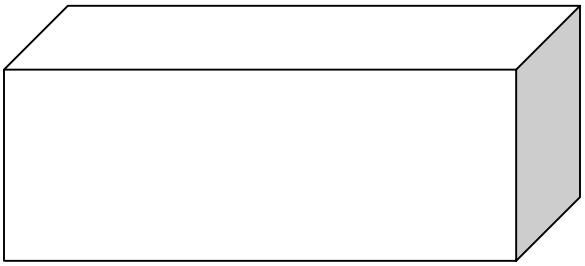
ANSWER KEY

Journal Entry

Name _____
Class _____
Date _____

Directions: Your best friend was absent from school today. The teacher has asked you to explain surface area and volume to your friend. Be sure to include all pertinent information, including definitions, formulas, and examples.

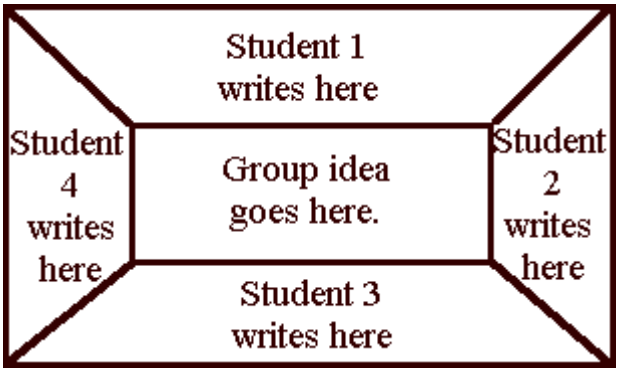
Answer will vary but should include the definitions of surface area (sum of area of faces) and volume (amount of space occupied by a 3D figure). The formulas to find each should be included ($SA = 2[lw + wh + lh]$ for surface area and $V = lwh$ for volume). The student should also give at least one example of each. For example:

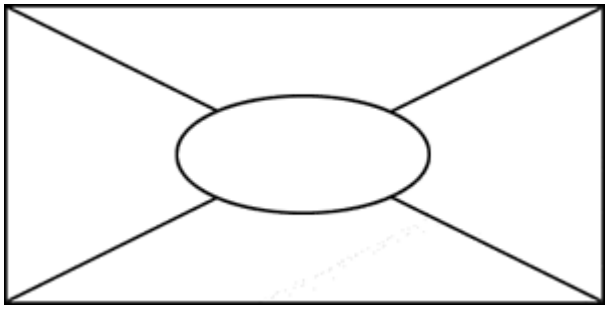


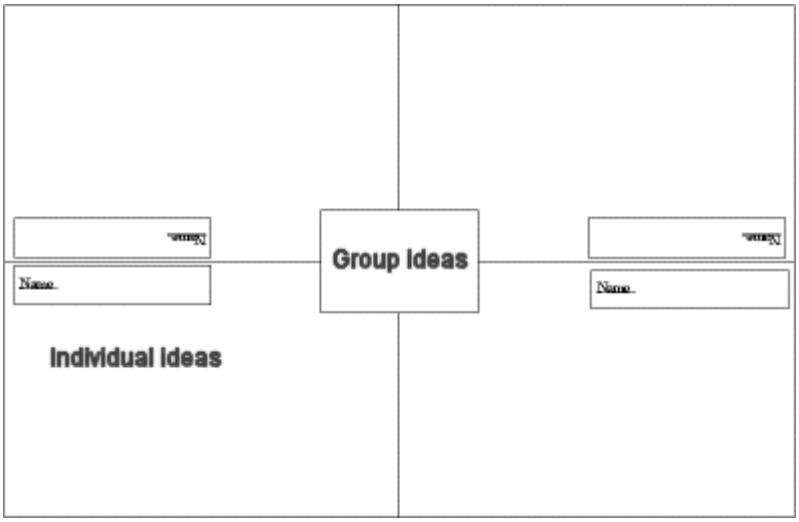
$SA = 2[lw + hw + lh]$
$SA = 2[12.5 \bullet 2 + 6.5 \bullet 2 + 12.5 \bullet 6.5]$
$SA = 2(119.25)$
$SA = 238.5 \text{ cm}^2$

$V = lwh$
$V = 12.5 \bullet 2 \bullet 6.5$
$V = 162.5 \text{ cm}^3$

Here are a few examples of different placemats:







Project Worksheet

Name _____
Class _____
Date _____

PACK IT UP!

Directions: Students will complete the table and answer the related questions. For the table, you do not have to use all of the rows.

Length	Width	Height	Surface Area	Volume

1. What do you notice about the volume of all the rectangular prisms?

_____

2. What do you notice about the surface areas of the rectangular prisms?

_____

3. What are the dimensions that give the greatest and the least surface area?

_____

4. As the design engineer, which box would the most cost effective in which to package the product? Justify your answer.

_____
_____
_____
_____

ANSWER KEY Note: These are the main combinations. The combinations may repeat making a different shaped box, but will have the same surface area and volume as the main combinations below (for example, a box with a length of 2, width of 3, and height of 4 will produce the same surface area and volume of a box with a length of 4, width of 3, and height of 2, although the boxes would not look the same).

Project Worksheet

Name _____
Class _____
Date _____

PACK IT UP!

Directions: Students will complete the table and answer the related questions. For the table, you do not have to use all of the rows.

Length	Width	Height	Surface Area	Volume
1	1	36	146	36
1	2	18	112	36
1	3	12	102	36
1	4	9	98	36
1	6	6	96	36
2	2	9	80	36
2	3	6	72	36
3	3	4	66	36

1. What do you notice about the volume of all the rectangular prisms?
_____There are all 36 _____

2. What do you notice about the surface areas of the rectangular prisms?
As the measurements get closer together (close to a cube), the surface area gets smaller. (answers will vary)

3. What are the dimensions that give the greatest and the least surface area?__1 by 1 by 36 – greatest and 3 by 3 by 4 – least _____

4. As the design engineer, which box would the most cost effective in which to package the product? Justify your answer.

The box that has the smallest surface area (3,3,4) would be the best because it requires less material to make, thus minimizing the cost to the company (answers will vary).

Assessment: Movie Theater Popcorn Name _____
Date _____

Situation: You are working for a movie theater and the manager has put you in charge of choosing which popcorn boxes to order. The two choices of large popcorn boxes are shown below:

Dimensions: 6″L x 4″W x 3″H

Dimensions: 4″L x 2″W x 9″H

1. When you are thinking about which box to order, what factors should you consider?

2. Which box looks like it would hold more popcorn? Why?

3. Determine which box would hold more popcorn. Explain how you arrived at your answer.

4. The cost of the box depends on how much material is needed to make the box. Which box would be cheaper for the manager to order? Explain how you arrived at your answer.

5. Having considered all of the factors, which box would you recommend that the manager order? Justify your answer.

ANSWER KEY

Assessment: Movie Theater Popcorn

Name _____

Date _____

Situation: You are working for a movie theater and the manager has put you in charge of choosing which popcorn boxes to order. The two choices of large popcorn boxes are shown below:

Dimensions: 6″L x 4″W x 3″H

Dimensions: 4″L x 2″W x 9″H

1. When you are thinking about which box to order, what factors should you consider? How much popcorn it can hold, the cost of each box, which one the customers would rather hold during the movie, etc.

2. Which box looks like it would hold more popcorn? Why?
   Various answers, but most likely the student will say the box on the left because it looks "fatter".

3. Determine which box would hold more popcorn. Explain how you arrived at your answer.

   Left box:
   $V = lwh$
   $V = 6 \bullet 4 \bullet 3$
   $V = 72 \text{ in.}^3$

   Right box:
   $V = lwh$
   $V = 4 \bullet 2 \bullet 9$
   $V = 72 \text{ in.}^3$

   Both of the boxes will hold the same amount of popcorn since their volumes are equal.

4. The cost of the box depends on how much material is needed to make the box. Which box would be cheaper for the manager to order? Explain how you arrived at your answer.

Left Box:
SA = 2[lw + wh + lh]
SA = 2[6•4 + 4•3 + 6•3]
SA = 2[24 + 12 + 18]
SA = 2(54)
SA = 108 in.^{2}

Right Box:
SA = 2[lw + wh + lh]
SA = 2[4•2 + 2•9 + 4•9]
SA = 2[8 + 18 + 36]
SA = 2(62)
SA = 124 in.^{2}

The box on the left would be cheaper for the theater owner to buy because it has the smaller surface area.

5. Having considered all of the factors, which box would you recommend that the manager order? Justify your answer.

The owner should purchase the box on the left. This is because they both have the same volume, meaning they can hold the same amount of popcorn, but the box on the left is cheaper to purchase.

Quiz: Nets, Surface Area, and Volume Name _____

Class _____

Date _____

A. Selected Response

1. A pentagonal prism has how many edges?

A) 5 B) 10 C) 15 D) 20

2. How many different pairs of faces are congruent in a rectangular prism that is not a cube?

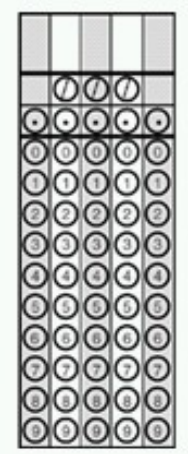
F) 1 G) 2 H) 3 I) 4

3. A hexagonal pyramid has how many vertices?

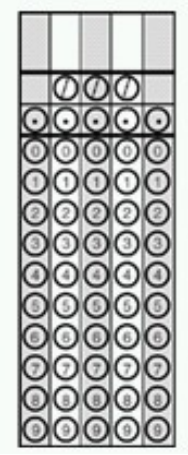
A) 5 B) 7 C) 12 D) 14

B. Student Produced Response

4. Determine the surface area in square inches of a rectangular prism with dimensions 4 in. by 3 in. by 2 in.



5. Determine the volume in cubic centimeters of a rectangular prism with dimensions 8 cm by 5 cm by 2 cm.



6. Make a net of square pyramid.

C. Brief Constructed Response

7. Sandy is filling her swimming pool. The pool is in the shape of a rectangular prism and is 30 feet long, 15 feet wide, and 8 feet deep.

Part A: Sandy wants to leave the top two feet of the pool unfilled to prevent spilling. How much water will it take to fill the pool in this manner? _____

Part B: Use what you know about volume to explain how you arrived at your answer.

_____

_____

_____

D. Extended Constructed Response

8. A company is trying to decide between two box designs. The dimensions of box A are 6 in. by 4 in. by 2 in. The dimensions of box B are 8 in. by 2 in. by 3 in.

Part A: Which box can be constructed with the least amount of material?

_____

Part B:

- Use what you know about surface area to explain how you arrived at your answer.

_____

_____

_____

- Could you construct another box with the same volume as your answer in Part A, but with a different surface area?

_____

_____

_____

ANSWER KEY

Quiz: Nets, Surface Area, and Volume

Name _____
Class _____
Date _____

A. Selected Response

1. A pentagonal prism has how many edges?

A) 5 B) 10 C) 15 D) 20

2. How many different pairs of faces are congruent in a rectangular prism that is not a cube?

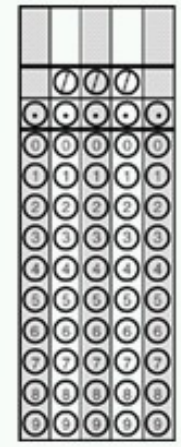
F) 1 G) 2 H) 3 I) 4

3. A hexagonal pyramid has how many vertices?

A) 5 B) 7 C) 12 D) 14

B. Student Produced Response

4. Determine the surface area in square inches of a rectangular prism with dimensions 4 in. by 3 in. by 2 in.



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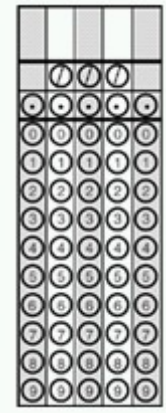
5. Determine the volume in cubic centimeters of a rectangular prism with dimensions 8 cm by 5 cm by 2 cm.



80

6. Make a net of square pyramid.

See net of square pyramid on earlier sheet

C. Brief Constructed Response

7. Sandy is filling her swimming pool. The pool is in the shape of a rectangular prism and is 30 feet long, 15 feet wide, and 8 feet deep.

Part A: Sandy wants to leave the top two feet of the pool unfilled to prevent spilling. How much water will it take to fill the pool in this manner?
2700 cubic feet

Part B: Use what you know about volume to explain how you arrived at your answer.
Since she is leaving 2 feet from the top empty, the actual height of the water will be 6 feet, not 8 feet. Using the formula to find the volume of a rectangular prism, $V = lwh$, I substituted the numbers 30 for length, 15 for width, and 6 for the height. Multiplying the numbers gives a result of 2700 cubic feet.

E. Extended Constructed Response

8. A company is trying to decide between two rectangular box designs. The dimensions of box A are 6 inches by 4 inches by 2 inches. The dimensions of box B are 8 in. by 2 in. by 3 in.

Part A: Which box can be constructed with the least amount of material?
Box A

Part B:

- Use what you know about surface area to explain how you arrived at your answer.

Box A:
$SA = 2[lw + wh + lh]$
$SA = 2[6 \bullet 4 + 4 \bullet 2 + 6 \bullet 2]$
$SA = 2(44)$
$SA = 88 \text{ in}^2$

Box B:
$SA = 2[lw + wh + lh]$
$SA = 2[8 \bullet 2 + 2 \bullet 3 + 8 \bullet 3]$
$SA = 2(46)$
$SA = 92 \text{ in}^2$

- Could you construct another box with the same volume as your answer in Part A, but with a different surface area?

Yes, you could construct a box with dimensions of 4 in., 4 in., and 3 in. The volume of this box would still be 48 in.^2, but the new surface area would be 80 in.^2.

$V = lwh$
$V = 4 \bullet 4 \bullet 3$
$V = 48 \text{ in.}^2$

$SA = 2[lw + wh + lh]$
$SA = 2[4 \bullet 4 + 4 \bullet 3 + 4 \bullet 3]$
$SA = 2[16 + 12 + 12]$
$SA = 2[40]$
$SA = 80 \text{ in.}^2$